1.6.2 Envelope Correlation as a Function of Antenna Separation

Since $\omega_m \tau = 2\pi v \tau/\lambda$, Eq. (1.6-15) can be regarded as a function of spatial separation $\zeta = v\tau$. We can now abandon the artificial assumption of a moving base station, and instead consider that $L_e(\zeta)$ gives the correlation between the envelopes of signals received simultaneously on two antennas at the base, separated by a distance ζ. To third order in k, Eq. (1.6-15) is directly analogous to Eq. (1.3-50) giving the auto-covariance of the E_z field seen at the mobile. Comparing arguments, we see that the base antenna separation must be a factor $(k \sin \xi)^{-1}$ times greater than that at the mobile to obtain the same correlation. Also, for $\xi = 0$ the third-order approximation gives a constant value of correlation independent of separation and equal to the value for $\zeta = 0$. Thus the fourth-order approximation is needed in this case. Estimates of the scattering circle diameter vary, but it seems obvious that it must be at least equal to the distance between buildings on opposite sides of the street where the mobile is located. This is substantiated by some experimental measurements.[19] Thus $2a$ might typically be 100 ft; at a range of $d = 2$ miles, $k = 0.005$; thus the power series expansion in k appears justified.

Curves of the correlation coefficient $\rho_e \doteq J_0^2(z_1)J_0^2(z_2)$ for $k = 0.006$ are shown in Figure 1.6-2, along with some values measured at 836 MHz.[22] Comparison with Figure 1.3-6 illustrates how much more rapidly the signals at the mobile become decorrelated with antenna separation. It should be emphasized that the model used here assumes *no* scatterers in the immediate vincity of the base station; the presence of even a small number of local scatterers would have a strong effect on the correlation, particularly for $\xi = 0$.

The model also does not include the direction of motion of the mobile with respect to the line-of-sight to the base station. One would expect that motion along the line-of-sight would require greater base station antenna separation for the same correlation, compared to motion perpendicular to the line-of-sight. This effect could be included by assuming that the scatterers lie on an ellipse with major axis along the direction of motion. A refined model of this type would approach the actual disposition of the scatterers more closely.

1.7 LABORATORY SIMULATION OF MULTIPATH INTERFERENCE

The testing of mobile radio transmission techniques in the field is time-consuming and often inconclusive, due to uncertainty in the statistical signal variations actually encountered. Laboratory testing with signals that duplicate the assumed statistical properties of the signals encountered in

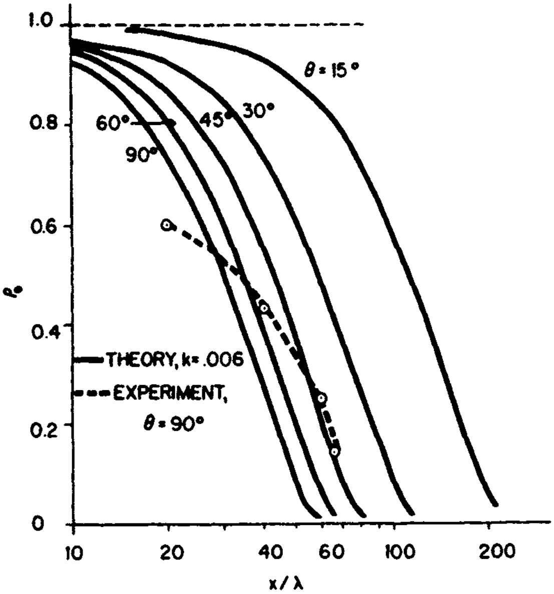

Figure 1.6-2 Correlation coefficient ρ_e between signals received on two antennas at a base station versus their separation and orientation angle θ.

the field is an attractive alternative, provided that all of the relevant properties can be simulated. Past approaches to the problem of simulating fading signals may be divided into three classes. First, tape recordings of the actual fading signals may be used.[23] In another method[24] a steady signal is split into several paths, each of which is then randomly phase modulated as shown in Figure 1.7-1(*a*). Uniformly distributed phase modulation is obtained by appropriately shaping the amplitude distribution of low-pass Gaussian noise. An approximation to Rayleigh fading is obtained by adding several such paths together. Frequency selective fading can also be produced by including path delay. However, the power spectrum of the output signal is very difficult to calculate or control. A third method[25] provides uniform phase modulation and Rayleigh envelope fading by amplitude modulation of the in-phase and quadrature components of a steady carrier with uncorrelated low-pass Gaussian noises, as shown in Figure 1.7-1(*b*). Frequency selective fading may be produced by combining several delayed fading signals. The different noise sources must have the same power spectrum to produce stationary fading, and the power spectrum of the fading signal will then be the same as the noise spectrum. The limitation with this approach is that only rational forms of the fading spectrum can be produced, whereas the spectra encountered in mobile radio are generally nonrational, as shown by Eqs. (1.2-11)–(1.2-13). A method[26] to simulate mobile radio fading that produces random phase modulation, a Rayleigh fading envelope, and a time-averaged, discrete approximation to the desired power spectrum will be discussed in the remainder of this section.

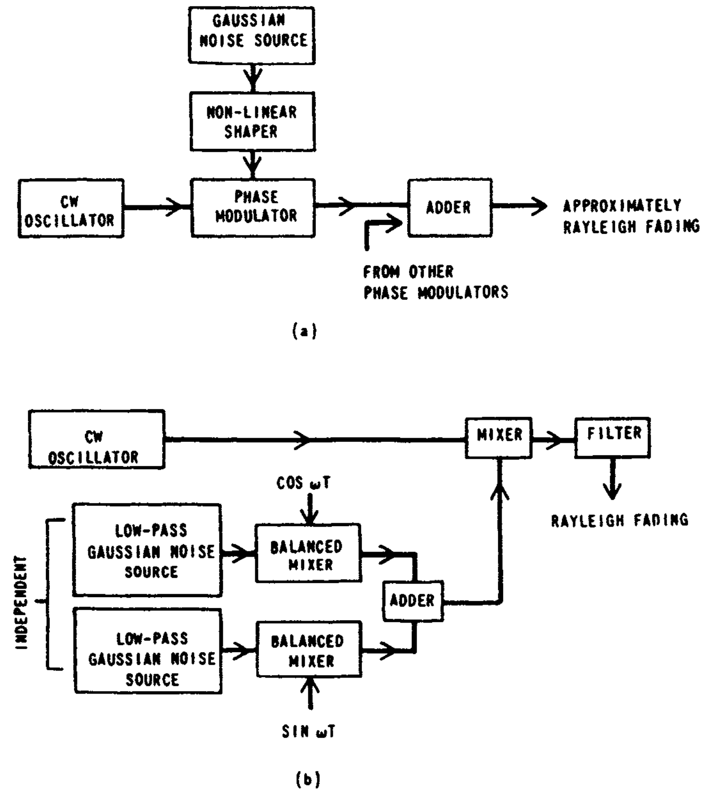

Figure 1.7-1 Two types of fading simulators. (*a*) Simulator using uniform phase modulation. (*b*) Simulator using quadrature amplitude modulation.

1.7.1 Mathematical Development

We start with an expression that represents the field as a superposition of plane waves:

$$E(t)=\mathrm{Re}\left[T(t)e^{i\omega_c t}\right], \tag{1.7-1}$$

where

$$T(t)=E_0\sum_{n=1}^{N} c_n e^{i(\omega_m t\cos\alpha_n+\phi_n)}, \qquad (1.7\text{-}2)$$

and

$$c_n^2=p(\alpha_n)\,d\alpha=\frac{1}{2\pi}\,d\alpha.$$

We assume that the arrival angles are uniformly distributed with $d\alpha=2\pi/N$; thus $c_n^2=1/N$, and

$$\alpha_n=\frac{2\pi n}{N}, \qquad n=1,2,\ldots,N. \qquad (1.7\text{-}3)$$

We further let $N/2$ be an odd integer; then the series can be rearranged to give

$$T(t)=\frac{E_0}{\sqrt{N}}\left\{\sum_{n=1}^{N/2-1}\left[e^{i(\omega_m t\cos\alpha_n+\phi_n)}+e^{-i(\omega_m t\cos\alpha_n+\phi_{-n})}\right] + e^{i(\omega_m t+\phi_N)}+e^{-i(\omega_m t+\phi_{-N})}\right\}. \qquad (1.7\text{-}4)$$

The first term in the sum represents waves with Doppler shifts that progress from $+\omega_m\cos(2\pi/N)$ to $-\omega_m\cos(2\pi/N)$ as n runs from 1 to $N/2-1$, while the Doppler shifts in the second term go from $-\omega_m\cos(2\pi/N)$ to $+\omega_m\cos(2\pi/N)$. Thus the frequencies in these two terms overlap. The third and fourth terms represent waves with the maximum Doppler shift of $+\omega_m$ and $-\omega_m$, respectively. Without much loss of generality it will be convenient to represent the signal in terms of waves whose frequencies do not overlap:

$$T(t)=\frac{E_0}{\sqrt{N}}\left\{\sqrt{2}\sum_{n=1}^{N_0}\left[e^{i(\omega_m t\cos\alpha_n+\phi_n)}+e^{-i(\omega_m t\cos\alpha_n+\phi_{-n})}\right] + e^{i(\omega_m t+\phi_N)}+e^{-i(\omega_m t+\phi_{-N})}\right\}, \qquad N_0=\frac{1}{2}\left(\frac{N}{2}-1\right) \qquad (1.7\text{-}5)$$

where the factor $\sqrt{2}$ has been used so that the total power in $E(t)$ will be unchanged. The simulation should, among other things, provide a good approximation to Rayleigh fading. If N is large enough we may invoke the Central Limit Theorem to conclude that $T(t)$ is approximately a complex Gaussian process, so that $|T|$ is Rayleigh as desired. From the work of Bennett[27] and Slack[28] it follows that the Rayleigh approximation is quite good for $N > 6$, with deviations from Rayleigh confined mostly to the extreme peaks. Further information as to the value of N may be obtained by examining the autocorrelation of $E(t)$:

$$R(\tau) = \langle E(t)E(t+\tau)\rangle$$
$$= \tfrac{1}{2}\operatorname{Re}\left[\langle T(t)T(t+\tau)e^{i\omega_c(2t+\tau)}\rangle + \langle T^*(t)T(t+\tau)e^{i\omega_c\tau}\rangle\right]. \quad (1.7\text{-}6)$$

The expectations are taken over the random phases ϕ_n, ϕ_m, and they occur only as sums of differences. The only terms that contribute are those involving $\phi_n - \phi_m$ with $n = m$, so that

$$R(\tau) = \frac{b_0}{N}\cos\omega_c\tau\left[4\sum_{n=1}^{N_0}\cos\left(\omega_m\tau\cos\frac{2\pi n}{N}\right) + 2\cos(\omega_m\tau)\right]. \quad (1.7\text{-}7)$$

We note that Eq. (1.7-7) is of the form of a carrier factor multiplied by a low-frequency factor:

$$R(\tau) = g(\tau)\cos\omega_c\tau. \quad (1.7\text{-}8)$$

We also know, from Eq. (1.3-7), that for a uniformly scattered field $g(\tau) = b_0 J_0(\omega_m\tau)$. Although this expression was derived for a continuum of arrival angles, we may suspect that if N is large enough, the quantity in brackets in Eq. (1.7-7) will closely approximate $J_0(\omega_m\tau)$. Noting that $J_0(x)$ may be defined as

$$J_0(x) = \frac{2}{\pi}\int_0^{\pi/2}\cos(x\cos\alpha)\,d\alpha, \quad (1.7\text{-}9)$$

the bracketed factor of Eq. (1.7-7) may be put in the form of a discrete approximation (Riemann sum) to the integral (1.7-9). We thus expect that

$$2\sum_{n=1}^{N_0}\cos\left(\omega_m\tau\cos\frac{2\pi n}{N}\right) + \cos(\omega_m\tau) = \frac{N}{2}J_0(\omega_m\tau). \quad (1.7\text{-}10)$$

Evaluation of Eq. (1.7-10) for various values of $\omega_m\tau$ and N shows that the series gives $J_0(\omega_m\tau)$ to eight significant digits for $\omega_m\tau < 15$ with $N = 34$. The number of frequency components needed is thus $\frac{1}{2}(\frac{34}{2} - 1) = 8$. The simulation will thus produce an RF spectrum which is a discrete approximation to the form

$$\left[1-\left(\frac{f-f_c}{f_m}\right)\right]^{-1/2}.$$

1.7.2 Realization of the Method

The simulation technique is now clear: N_0 low-frequency oscillators with frequencies equal to the Doppler shifts $\omega_m \cos(2\pi n/N)$, $n=1,2,\ldots N_0$, plus one with frequency ω_m are used to generate signals frequency-shifted from

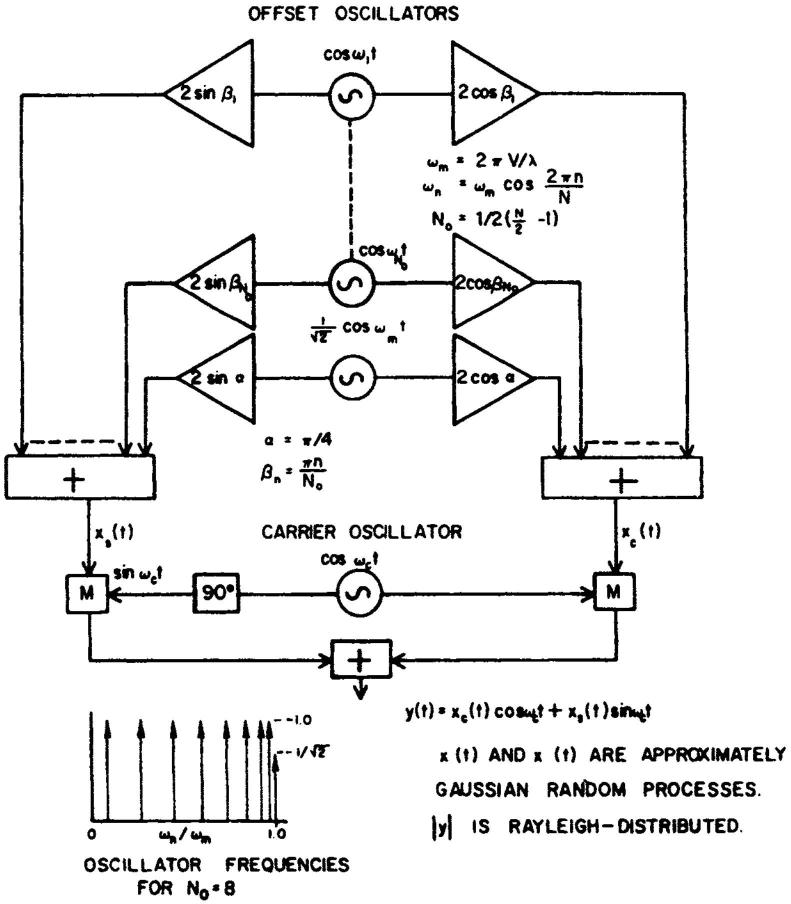

Figure 1.7-2 Simulator that duplicates mobile radio spectrum.

a carrier frequency ω_c using modulation methods. The amplitudes of all the components are made equal to unity except for the one with frequency ω_m, which is set equal to $1/\sqrt{2}$. The phases β_n are chosen appropriately so that the probability distribution of the resultant phase will be as close as possible to a uniform distribution, $1/2\pi$. A block diagram of such a simulator is shown in Figure 1.7-2 along with an illustration of the frequency spacings of the oscillators for $N_0=8$. By taking advantage of some trigonometric relationships, the proper oscillator phases are provided by amplifiers with gains set equal to $2\cos\beta_n$ or $2\sin\beta_n$. The outputs of the individual oscillators, with the appropriate gain factors, are first summed to produce in-phase (x_c) and quadrature (x_s) bands, which are then multiplied by in-phase and quadrature carrier components, respectively, and then summed to produce the final composite output signal $y(t)$. From the block diagram we get

$$x_c(t)=2\sum_{n=1}^{N_0}\cos\beta_n\cos\omega_n t+\sqrt{2}\cos\alpha\cos\omega_m t, \qquad (1.7\text{-}11)$$

$$x_s(t)=2\sum_{n=1}^{N_0}\sin\beta_n\cos\omega_n t+\sqrt{2}\sin\alpha\cos\omega_m t. \qquad (1.7\text{-}12)$$

The phase of $y(t)$ must be random and uniformly distributed from zero to 2π; this may be accomplished in several ways, provided $\langle x_c^2\rangle\approx\langle x_s^2\rangle$ and $\langle x_c x_s\rangle\approx 0$. We have

$$\langle x_c^2\rangle=2\sum_{n=1}^{N_0}\cos^2\beta_n+\cos^2\alpha$$

$$=N_0+\cos^2\alpha+\sum_{n=1}^{N_0}\cos 2\beta_n, \qquad (1.7\text{-}13)$$

$$\langle x_s^2\rangle=2\sum_{n=1}^{N_0}\sin^2\beta_n+\sin^2\alpha$$

$$=N_0+\sin^2\alpha-\sum_{n=1}^{N_0}\cos 2\beta_n, \qquad (1.7\text{-}14)$$

$$\langle x_c x_s\rangle=2\sum_{n=1}^{N_0}\sin\beta_n\cos\beta_n+\sin\alpha\cos\alpha. \qquad (1.7\text{-}15)$$

By choosing $\alpha=0$, $\beta_n=\pi n/(N_0+1)$, we find $\langle x_c x_s\rangle \equiv 0$ and $\langle x_c^2\rangle = N_0$, $\langle x_s^2\rangle = N_0+1$. (Note that the brackets denote time averages now.) Thus $y(t)$ is a narrow-band signal centered on a carrier frequency ω_c, having Rayleigh fading characteristics, and with autocorrelation function approximately equal to $J_0(\omega_m\tau)$. Its spectrum is therefore the nonrational form given by Eq. (1.2-4), corresponding to a uniform antenna pattern, $G(\alpha)=1$, and uniform distribution of the incident power, $p(\alpha)=1/2\pi$. Random FM is also produced by this method. Since the carrier frequency is provided by one oscillator, it may be set to some convenient value, say 30 MHz, and voice-modulated either in amplitude or frequency for use with various reception techniques. The performance of a simulator built with nine offset oscillators ($N_0=8$) is illustrated in Figures 1.7-3 to 1.7-6, showing measured cumulative distribution of the envelope, autocorrelation function, RF spectrum, and random FM power spectrum. Comparison with the expected Rayleigh distribution, Bessel function autocorrelation, and theoretical RF and random FM spectra shows excellent agreement.

This technique may be extended to provide up to N_0 independently fading signals while still using the same offset oscillators. The nth oscillator is given an additional phase shift $\gamma_{nj}+\beta_{nj}$, with gains as before. By imposing the additional requirement that the output signals $y_j(t)$ be uncorrelated (or as nearly so as possible), the appropriate values for γ_{nj} and β_{nj} can be determined. The choices are not unique, but the following seems to be the simplest:

$$\beta_{nj}=\frac{\pi n}{N_0+1}, \tag{1.7-16}$$

$$\gamma_{nj}=\frac{2\pi(j-1)}{N_0+1}, \qquad n=1,2,\ldots,N_0. \tag{1.7-17}$$

By using two quadrature low-frequency oscillators per offset in place of the single oscillators shown in Figure 1.7-2, the use of phase shifters to perform the $\gamma+\beta$ shift can be eliminated. This leads to modified amplifier gains as sketched in Figure 1.7-7 for the nth offset amplifier of the jth simulator. The $N=2$ curve in the $p(R)$ graph of Figure 1.7-3 shows the resulting combined envelope statistics of a simulated two-branch maximal ratio diversity combiner (cf. Section 5.2).

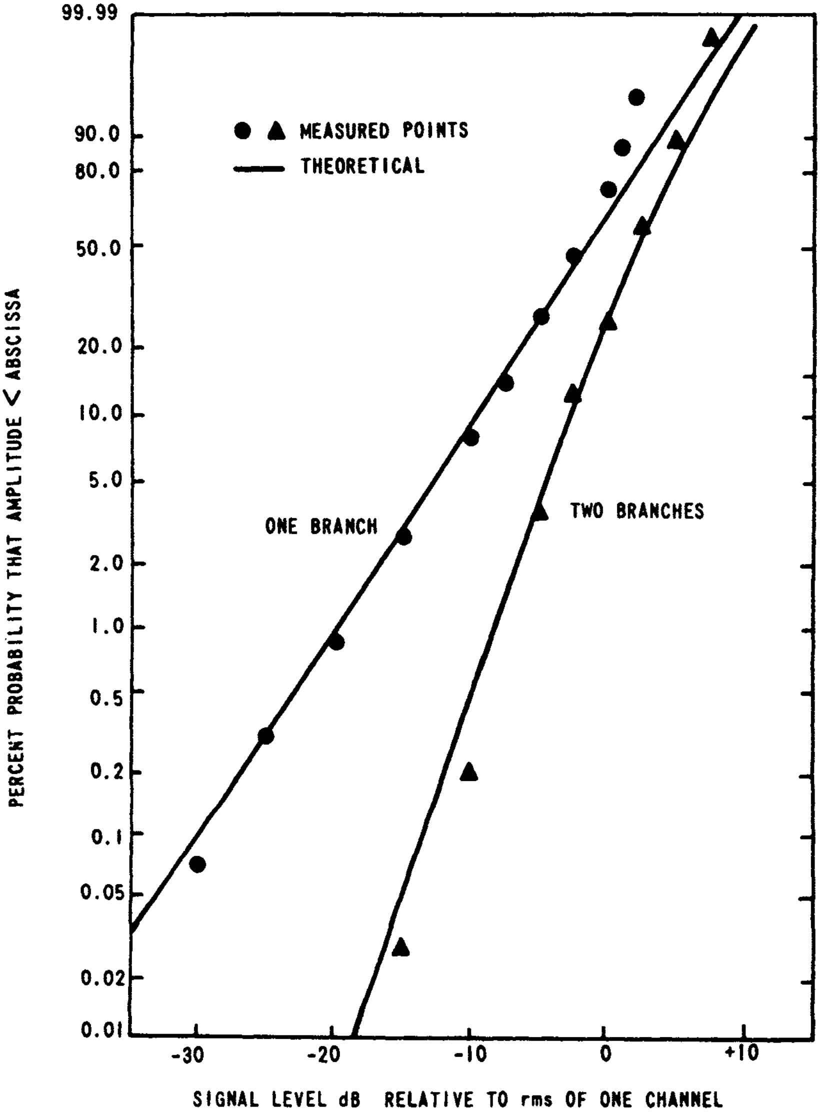

Figure 1.7-3 Probability distributions measured from the output of a fading simulator.

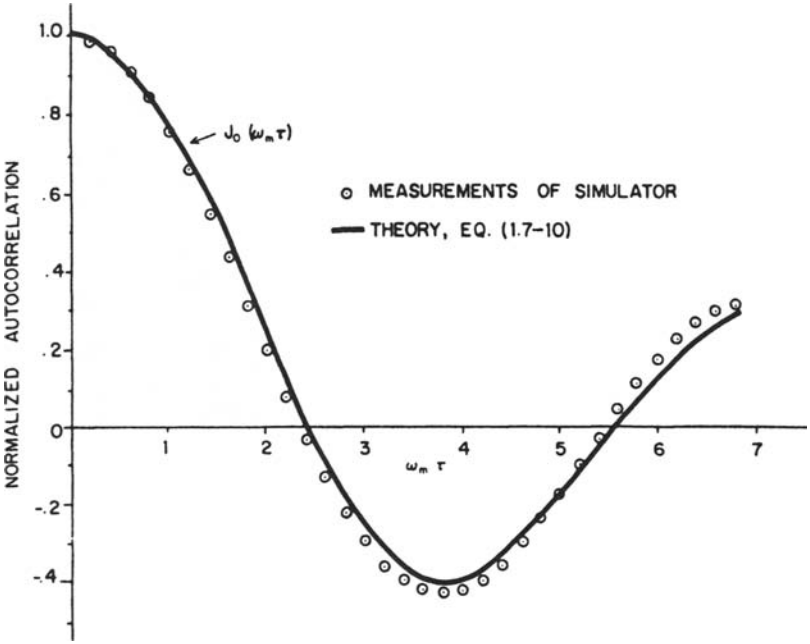

Figure 1.7-4 Comparison of theoretical autocorrelation function of the fading signal with data from a laboratory simulator.

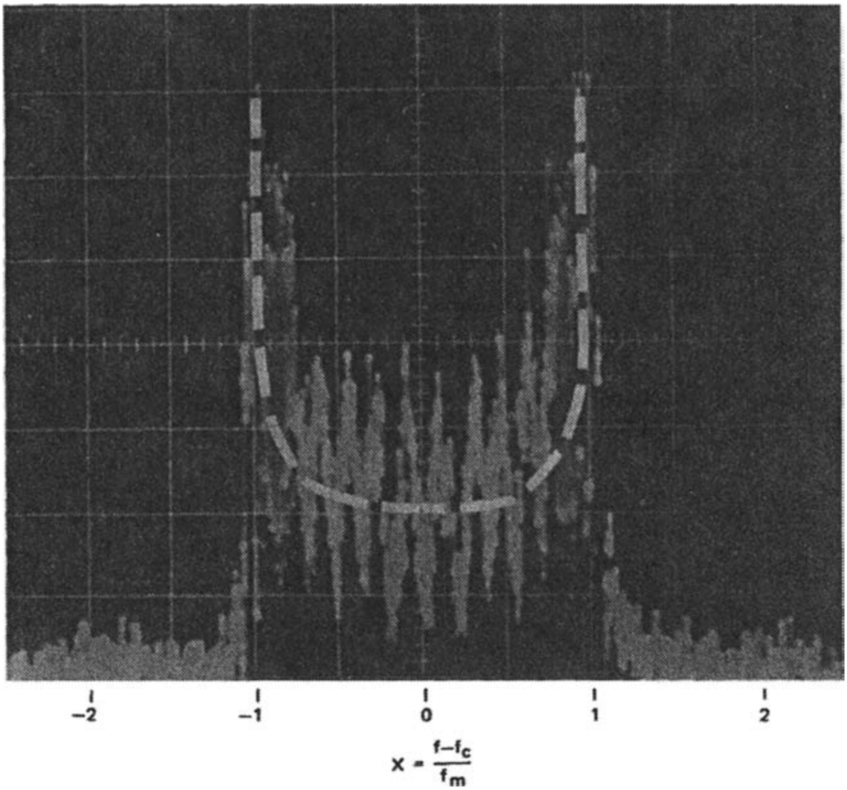

Figure 1.7-5 RF Spectrum of simulated fading carrier. Dashed line is the theoretical spectrum, $(1-X^2)^{-1/4}$.

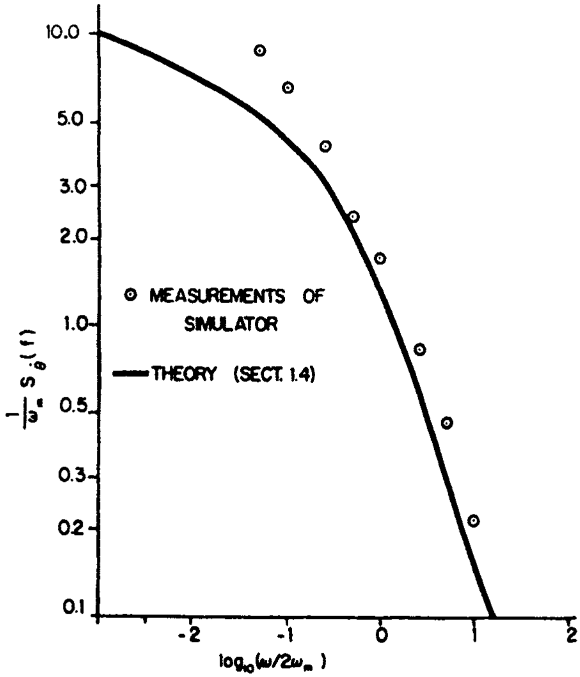

Figure 1.7-6 Comparison of theoretical spectrum of the instantaneous frequency with data from laboratory fading simulator.

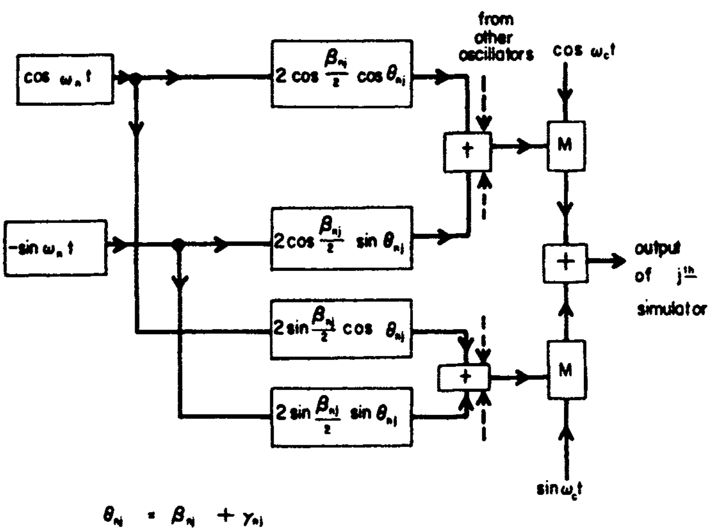

Figure 1.7-7 Use of quadrature low-frequency oscillators to provide uncorrelated fading carriers.

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