電子情報通信特論
Advanced Course of Electronics, Information and Communications

Fundamentals of MIMO

担当:金 ミンソク 准教授
mskim@eng.niigata-u.ac.jp

自然科学研究科　電気情報工学専攻

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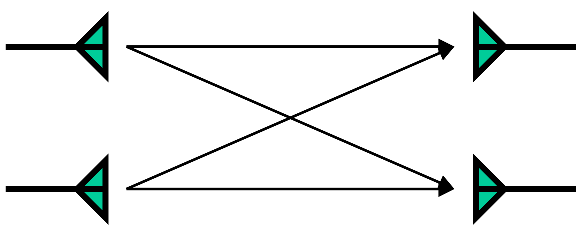
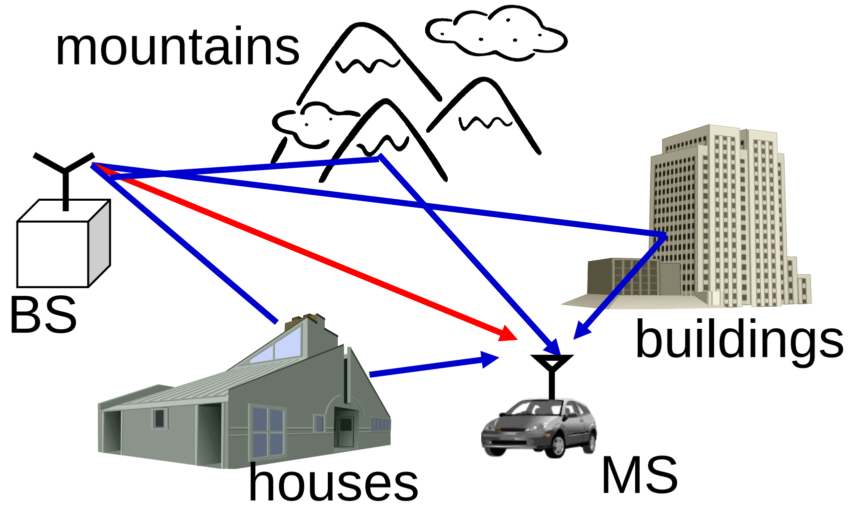
Why MIMO

- MIMO is an acronym that stands for Multiple-Input Multiple-Output
- Motivation: current wireless systems
  - Capacity constrained networks
  - Issues related to quality and coverage
- MIMO exploits the space dimension to improve wireless systems
  - Capacity, range and reliability



Multipath Channel

- In wireless communication the propagation channel is characterized by multipath propagation due to scattering on different obstacles
  - Temporal variation → Narrowband Fading (multipath fading)
  - Temporal dispersion → Wideband Fading (frequency selectivity fading)

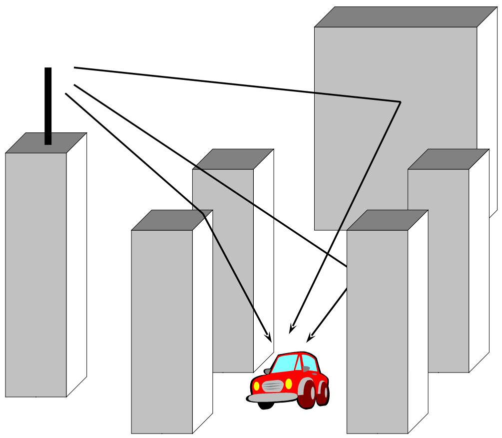


Multipath Environment

How MIMO Works

- MIMO takes advantage of multi-path
- MIMO uses multiple antennas to send multiple parallel signals
- With MIMO, the receiving end uses special signal processing to separate the multiple signals

Multipath Environment



MIMO transmission

→ Utilization of different propagation paths for different data stream

How MIMO Works

- Multiple transmit antennas were used to induce channel variations, which can then be exploited by opportunistic communication techniques
- MIMO techniques can provide both a *power gain* and a *degree-of-freedom gain*
- Degrees of freedom can be exploited by spatially multiplexing several data streams onto the MIMO channel, and lead to an increase in the capacity
- The multiple antennas allow spatial separation of the signals from the different users.
- The scattering environment is rich enough to allow the receive antennas to separate out the signals from the different transmit antennas.

Singular Value Decomposition of MIMO Channels

- Time invariant MIMO channel is assumed as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}$$

$$\boldsymbol{x} \in \mathbb{C}^{N_t}, \boldsymbol{y} \in \mathbb{C}^{N_r}, \boldsymbol{n} \sim CN\left(0, N_0 \boldsymbol{I}_{N_r}\right) \in \mathbb{C}^{N_r}, \boldsymbol{H} \in \mathbb{C}^{N_r \times N_t}$$

- The capacity can be computed by decomposing the vector channel into a set of parallel, independent scalar Gaussian sub-channels

$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H$$

$\boldsymbol{U} \in \mathbb{C}^{N_r \times N_r}$, $\boldsymbol{V} \in \mathbb{C}^{N_t \times N_t}$ unitary matrices

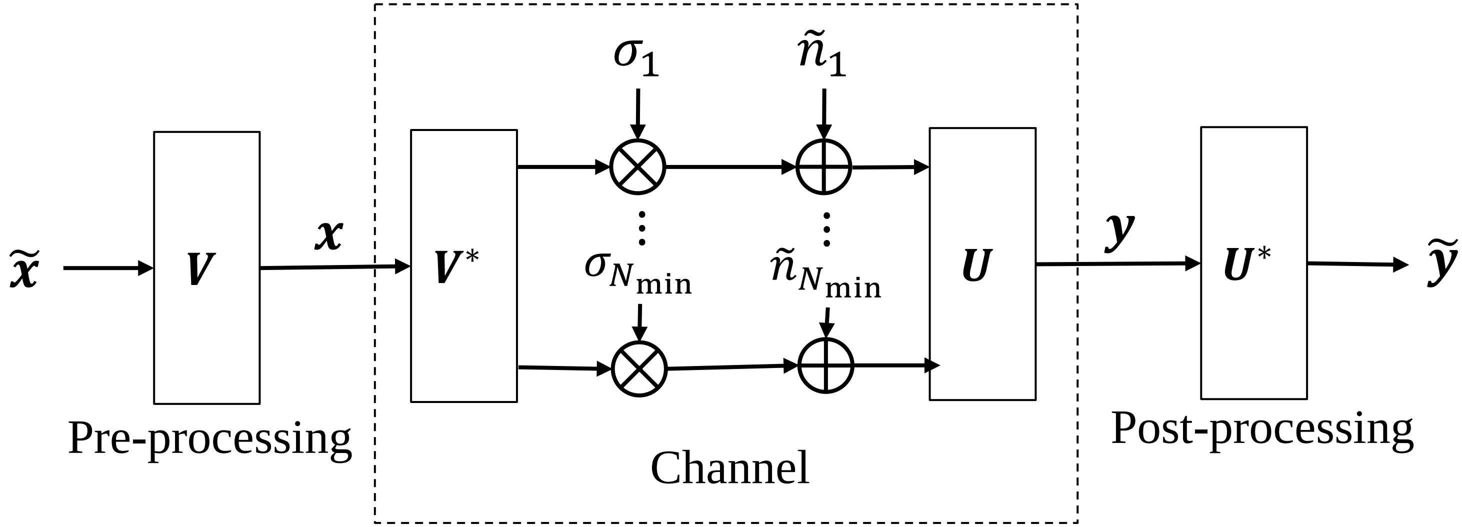
singular values

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{N_{\min}} \end{bmatrix} \in \mathbb{R}^{N_r \times N_t}$$

$$N_{\min} = \min(N_t, N_r)$$

Coordinate Transform Relation

- Converting the MIMO channel into a parallel channel through the SVD



$$\boldsymbol{U}^H\boldsymbol{y} = \boldsymbol{U}^H\boldsymbol{H}\boldsymbol{x} + \boldsymbol{U}^H\boldsymbol{n}$$
$$\boldsymbol{U}^H\boldsymbol{y} = \boldsymbol{\Sigma}\boldsymbol{V}^H\boldsymbol{x} + \boldsymbol{U}^H\boldsymbol{n}$$
$$\tilde{\boldsymbol{y}} = \boldsymbol{\Sigma}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{n}}$$

$\boldsymbol{V}^H\boldsymbol{x}$, $\boldsymbol{U}^H\boldsymbol{y}$, $\boldsymbol{U}^H\boldsymbol{n}$: coordinate transformations

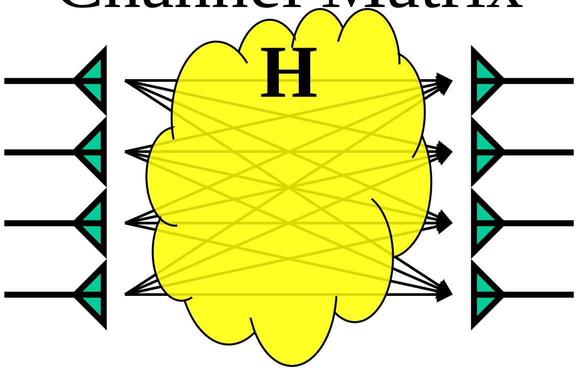
$$\boxed{\tilde{y}_i = \sigma_i\tilde{x}_i + \tilde{n}_i \text{ for } i = 1, \ldots, N_{\min}}$$

parallel, independent scalar Gaussian sub-channels

MIMO Eigen-mode Transmission

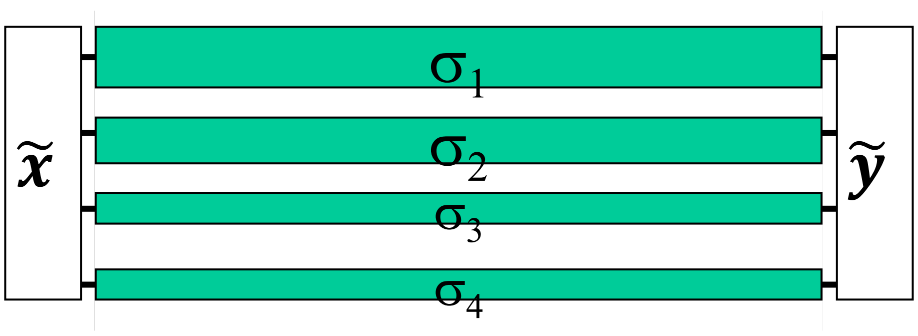
- Channel parallelization

Channel Matrix



$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H$$

$\boldsymbol{U} \in \mathbb{C}^{N_r \times N_r}$ unitary
$\boldsymbol{V} \in \mathbb{C}^{N_t \times N_t}$ matrices
$\boldsymbol{\Sigma}$ real diagonal matrix



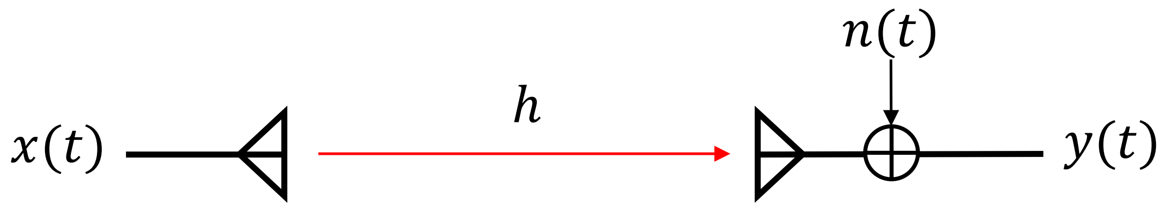
MIMO eigen-mode transmission

$$\tilde{\boldsymbol{y}} = \boldsymbol{U}^H\boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H\boldsymbol{V}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{n}}$$
$$= \boldsymbol{\Sigma}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{n}}$$

Isolation of parallel channels

SISO Transmission

- SISO (Single-Input-Single-Output) Model



$$y(t) = h \cdot x(t) + n(t)$$

$$\hat{x}(t) = h^{-1} \cdot y(t) = x(t) + h^{-1} \cdot n(t)$$

$$\overline{|x(t)|^2} = P_s$$

$$\overline{|n(t)|^2} = P_n$$

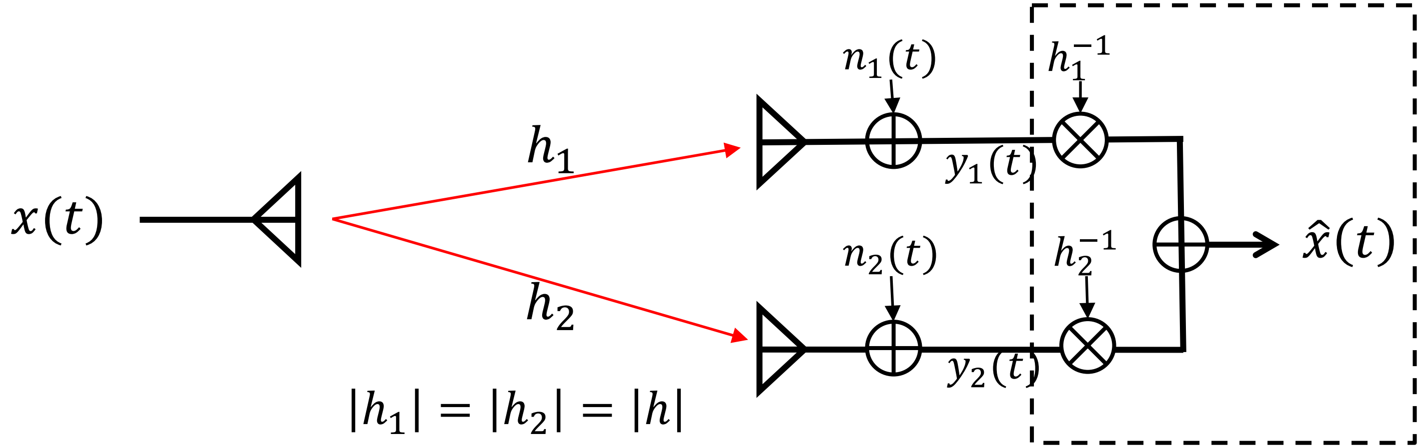
- Channel capacity

$$\boxed{C_{\mathrm{SISO}} = \log_2(\gamma_{\mathrm{SISO}} + 1) \ \ [\mathrm{bps/Hz}]}$$

$$\gamma_{\mathrm{SISO}} = |h|^2 \frac{P_s}{P_n}$$

SIMO Transmission

- SIMO (Single-Input-Multiple-Output) diversity



$$|h_1| = |h_2| = |h|$$

$$\hat{x}(t) = 2x(t) + h_1^{-1} \cdot n_1(t) + h_2^{-1} \cdot n_2(t)$$

$$\overline{|x(t)|^2} = P_s$$
$$\overline{|n_1(t)|^2} = \overline{|n_2(t)|^2} = P_n$$

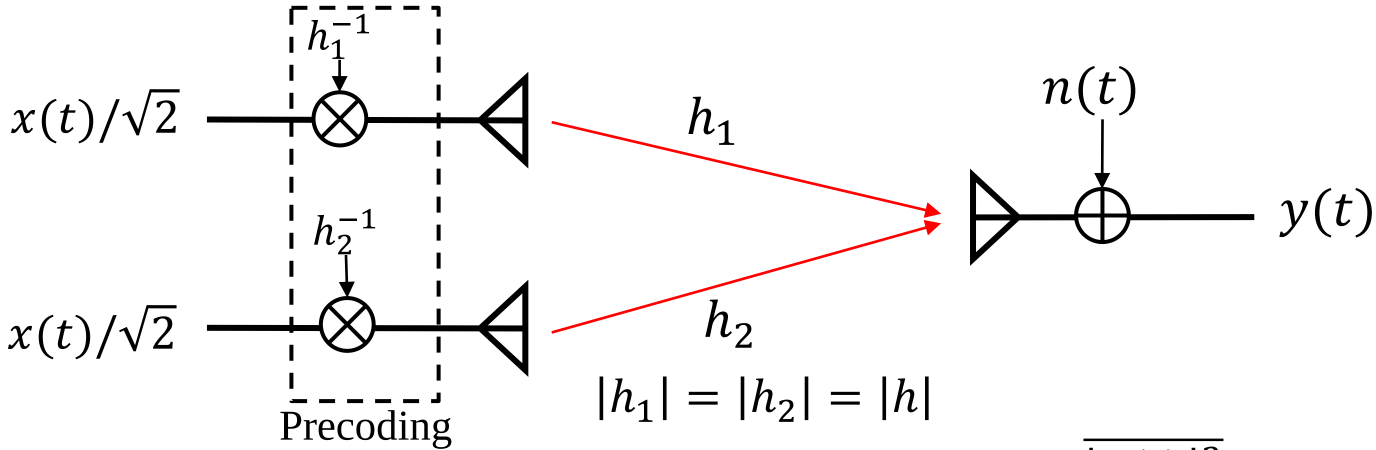
- Channel capacity

$$\gamma = |h|^2 \frac{2P_s}{P_n} = 2\gamma_{\mathrm{SISO}}$$

$$\boxed{C = \log_2(2\gamma_{\mathrm{SISO}} + 1) \ \ [\mathrm{bps/Hz}]}$$

MISO Transmission

- MISO (Multiple-Input-Single-Output) diversity



$$\overline{|x(t)|^2} = P_s$$
$$\overline{|n(t)|^2} = P_n$$

$$y(t) = \sqrt{2}|h|x(t) + n(t)$$

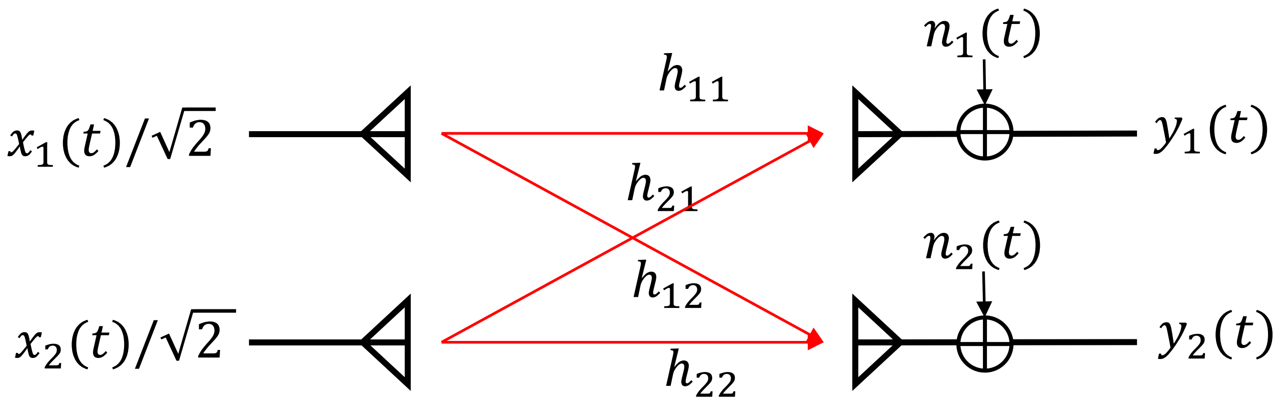
- Channel capacity

$$\gamma = |h|^2 \frac{2P_s}{P_n} = 2\gamma_{\text{SISO}}$$

$$\boxed{C = \log_2(2\gamma_{\text{SISO}} + 1) \ \ [\text{bps/Hz}]}$$

MIMO Transmission

- MIMO (Multiple-Input-Multiple-Output) Model



$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix} \quad \overline{|x_1(t)|^2} = \overline{|x_2(t)|^2} = P_s, \quad \overline{|n_1(t)|^2} = \overline{|n_2(t)|^2} = P_n$$

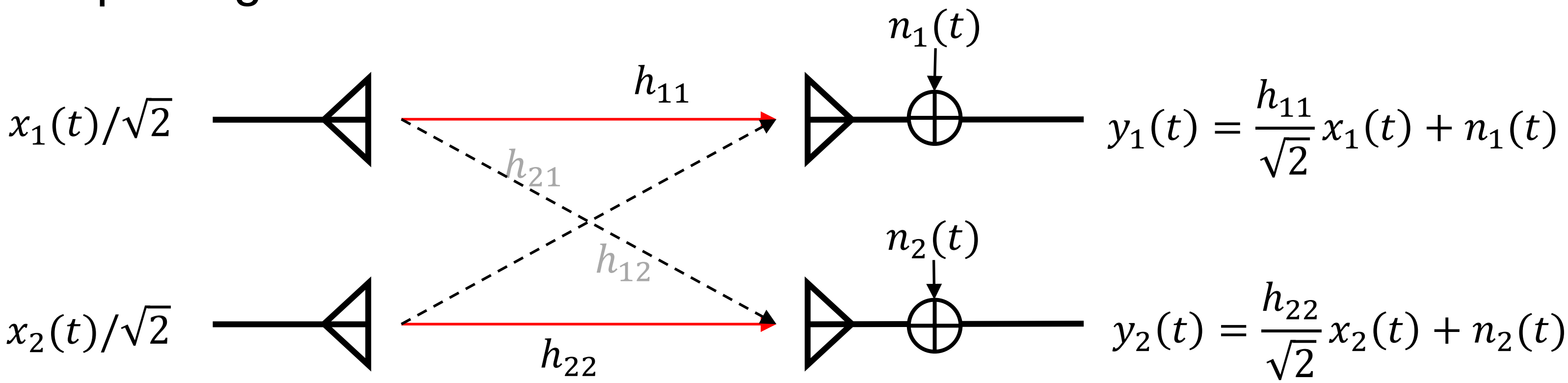
$$\boldsymbol{y}(t) = \boldsymbol{H}\boldsymbol{x}(t) + \boldsymbol{n}(t)$$

$$\hat{\boldsymbol{x}}(t) = \boldsymbol{H}^{-1}\boldsymbol{y}(t)$$

Inverse of channel matrix should exist (full rank)

Orthogonally Polarized MIMO

- Multiplexing



$$|h_{11}| = |h_{22}| = |h| \qquad |h_{12}| = |h_{21}| \approx 0$$

$$\hat{x}_1(t) = \frac{1}{\sqrt{2}} x_1(t) + h_{11}^{-1} n_1(t), \quad \hat{x}_2(t) = \frac{1}{\sqrt{2}} x_2(t) + h_{22}^{-1} n_2(t)$$

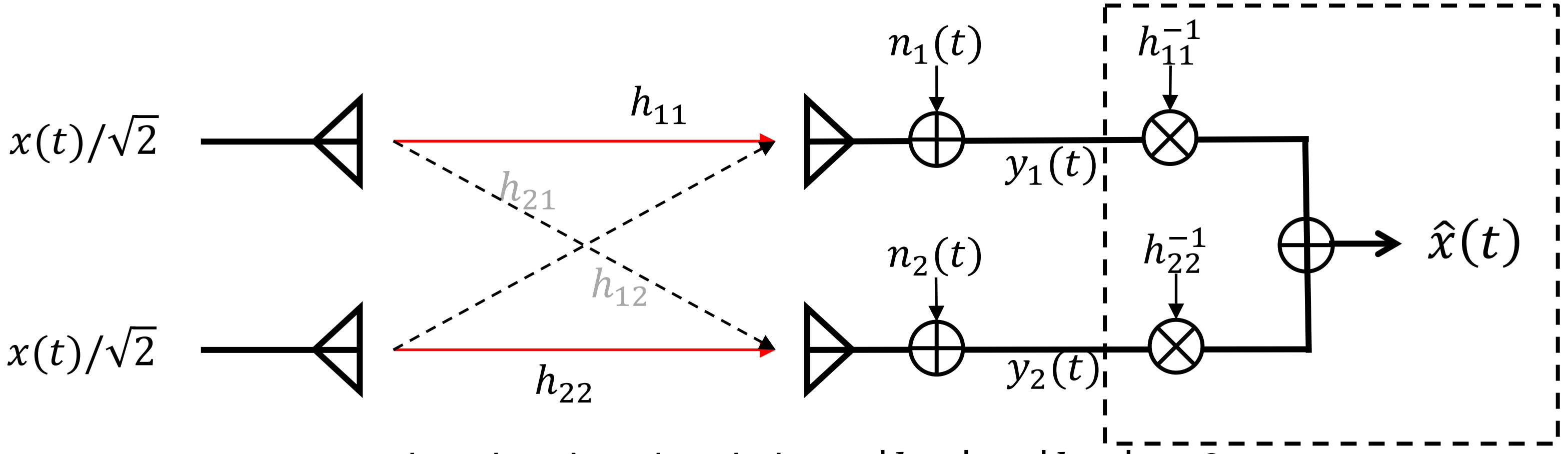
- Capacity

$$\gamma_1 = \gamma_2 = \frac{|h|^2}{2} \frac{P_s}{P_n} = \frac{1}{2} \gamma_{\text{SISO}}$$

$$C = \log_2(\gamma_1 + 1) + \log_2(\gamma_2 + 1) = \boxed{2 \log_2\left(\frac{1}{2}\gamma_{\text{SISO}} + 1\right) \text{ [bps/Hz]}}$$

Orthogonally Polarized MIMO

- Diversity



$$|h_{11}| = |h_{22}| = |h| \qquad |h_{12}| = |h_{21}| \approx 0$$

$$\hat{x}(t) = \sqrt{2}x(t) + h_{11}^{-1}n_1(t) + h_{22}^{-1}n_2(t)$$

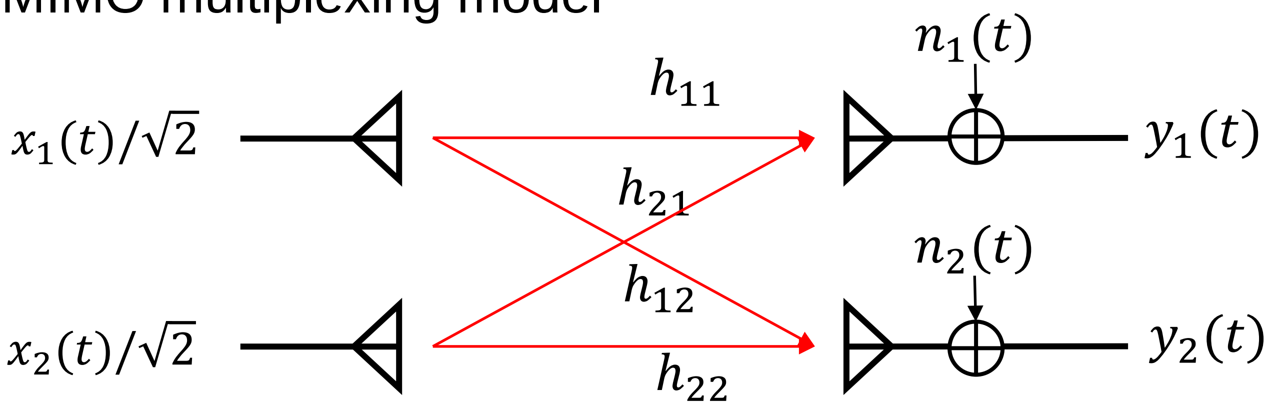
- Capacity

$$\gamma = \frac{2P_s}{2P_n/|h|^2} = \gamma_{\text{SISO}}$$

$$\boxed{C = \log_2(\gamma_{\text{SISO}} + 1) \ \text{[bps/Hz]}}$$

MIMO Transmission

- 2x2 MIMO multiplexing model



$$\overline{|x_1(t)|^2} = \overline{|x_2(t)|^2} = P_s$$
$$\overline{|n_1(t)|^2} = \overline{|n_2(t)|^2} = P_n$$

$$\boldsymbol{y}(t) = \boldsymbol{H}\boldsymbol{x}(t) + \boldsymbol{n}(t) = \frac{1}{\sqrt{2}}\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix}$$

$$\hat{\boldsymbol{x}}(t) = \boldsymbol{H}^{-1}\boldsymbol{y}(t) = \begin{bmatrix} \frac{1}{\sqrt{2}} x_1(t) + \frac{1}{\det \boldsymbol{H}}\left(h_{22}n_1(t) - h_{12}n_2(t)\right) \\ \frac{1}{\sqrt{2}} x_2(t) + \frac{1}{\det \boldsymbol{H}}\left(-h_{21}n_1(t) + h_{11}n_2(t)\right) \end{bmatrix}$$

MIMO Transmission

- Multiplexing

$$\gamma_1 = |\det \boldsymbol{H}|^2 \frac{P_s/2}{(|h_{22}|^2 + |h_{12}|^2)P_n} \qquad \gamma_2 = |\det \boldsymbol{H}|^2 \frac{P_s/2}{(|h_{21}|^2 + |h_{11}|^2)P_n}$$

$$C = \sum_{i=1}^{2} \log_2(\gamma_i + 1) \text{ [bps/Hz]}$$

- Examples

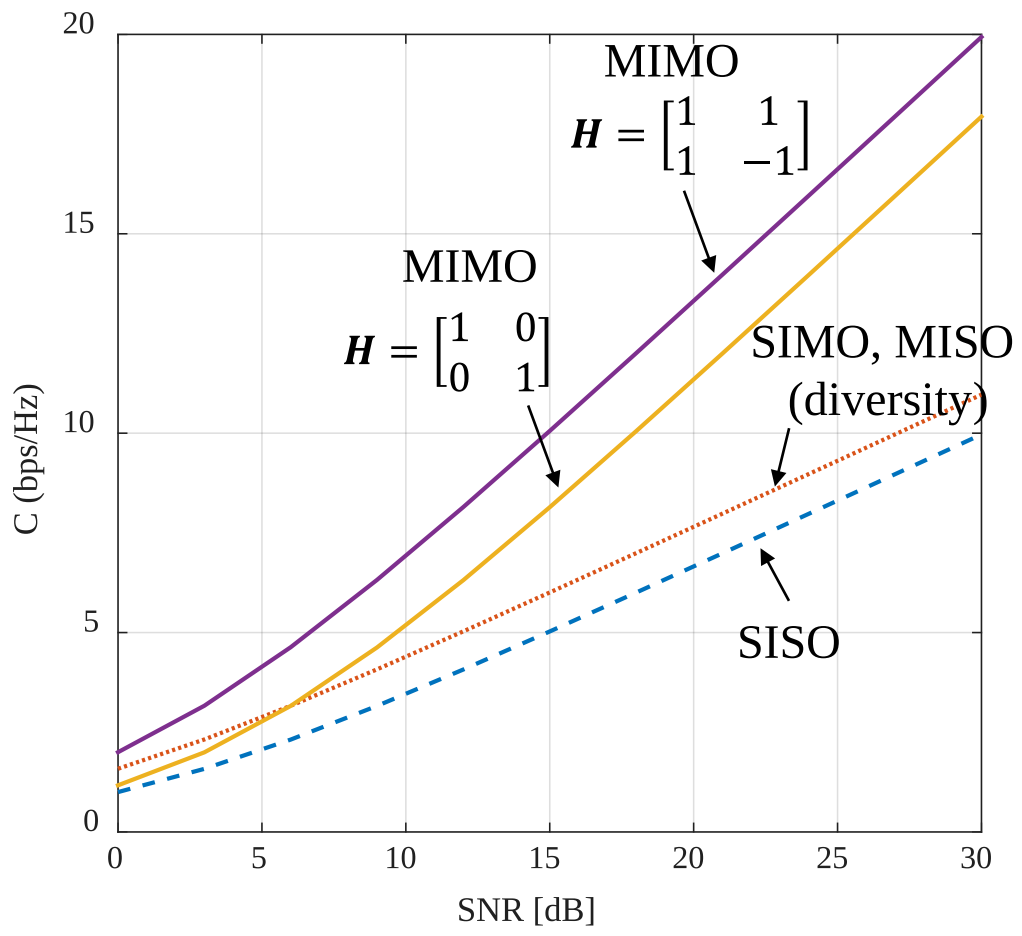
$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \gamma_1 = \gamma_2 = \frac{P_s}{2P_n} = \frac{1}{2}\gamma_{\text{SISO}} \qquad C = 2\log_2\left(\frac{1}{2}\gamma_{\text{SISO}} + 1\right)$$

$$\boldsymbol{H} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \gamma_1 = \gamma_2 = 0 \qquad C = 0$$

$$\boldsymbol{H} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \gamma_1 = \gamma_2 = 4\frac{P_s/2}{2P_n} = \gamma_{\text{SISO}} \qquad C = 2\log_2(\gamma_{\text{SISO}} + 1)$$

MIMO vs. SIMO/MISO

- Linear vs. Logarithmic Improvement



MIMO Channel Capacity

- System model

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}$$

where $\boldsymbol{x}$ is a multivariate random process which is circularly symmetric

- Channel capacity

$$C = \log\det\left(\frac{1}{P_n}\boldsymbol{H}\boldsymbol{R}_x\boldsymbol{H}^H + \boldsymbol{I}_{N_r}\right)$$

  - w/o CSIT → equal power transmission

$$C = E\left[\log\det\left(\frac{1}{N_t}\frac{P}{P_n}\boldsymbol{H}\boldsymbol{H}^H + \boldsymbol{I}_{N_r}\right)\right] = E\left[\sum_{i=1}^{N_{\min}} \log\left(\frac{\sigma_i^2}{N_t}\gamma_0 + 1\right)\right]$$

$$N_{\min} = \operatorname{rank}\boldsymbol{H} \qquad \gamma_0 = \frac{P}{P_n}$$

MIMO Channel Capacity (2)

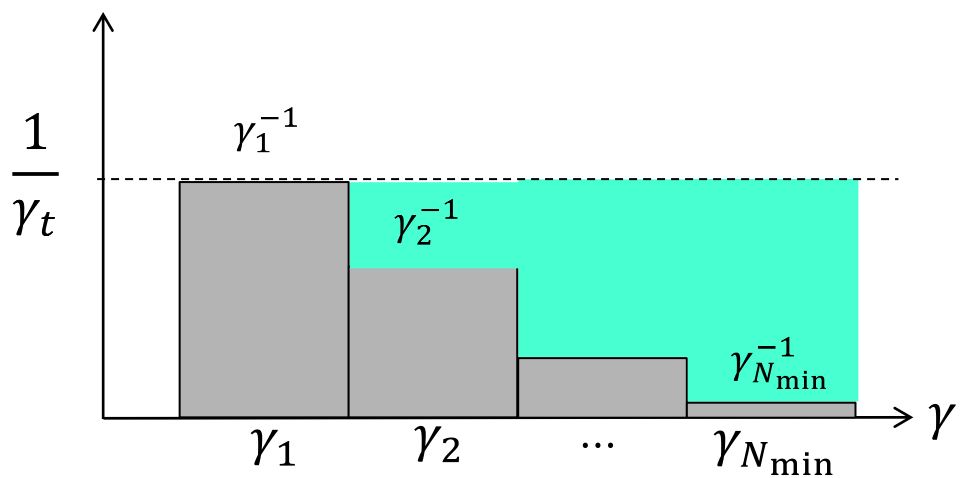
- w/ CSIT → water-filling power allocation

$$C\ [\text{bps/Hz}] = \max_{P_i:\sum_i P_i \le P} \sum_{i=1}^{N_{\min}} \log_2\left(1 + \frac{P_i}{P}\gamma_i\right) \qquad \gamma_i = \frac{\sigma_i^2 P}{P_n}$$

where $P_1, \dots P_{N_{\min}}$ are the water-filling power allocation

$$\frac{P_i}{P} = \max\left(\frac{1}{\gamma_t} - \frac{1}{\gamma_i}, 0\right) \qquad \text{Constraint: } \sum P_i \le P$$

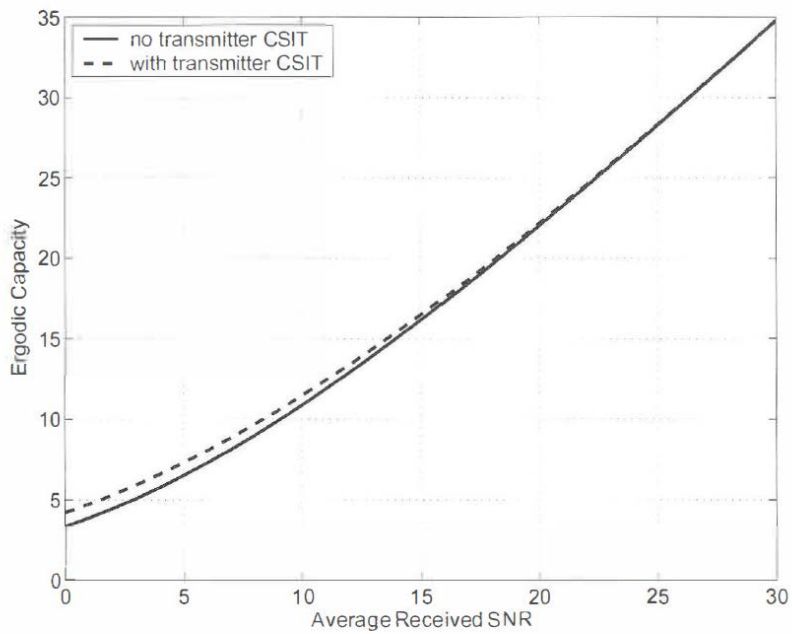
where $\gamma_i = \sigma_i^2 P / P_n$

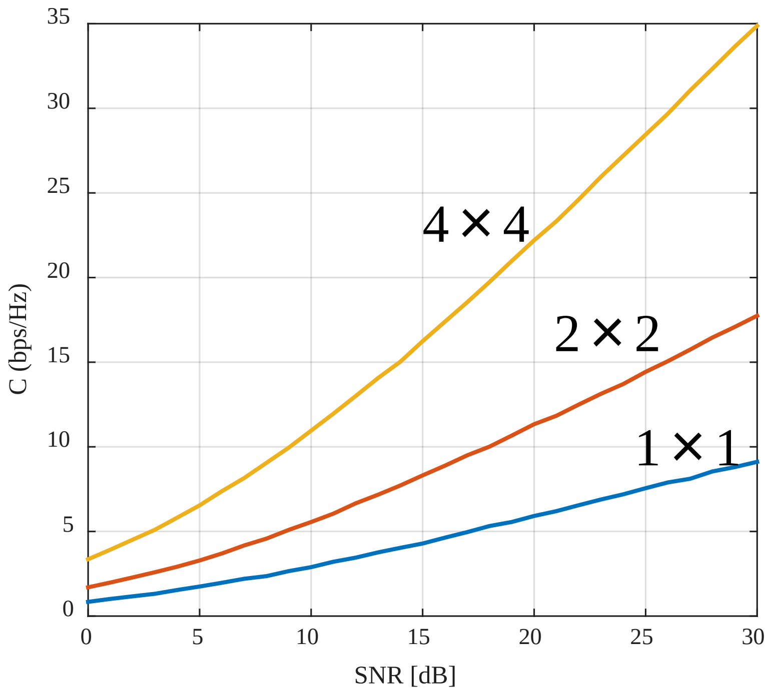


MIMO Channel Capacity (3)

- The ergodic capacity with i.i.d. (independent identical distributed) complex Gaussian channel gains

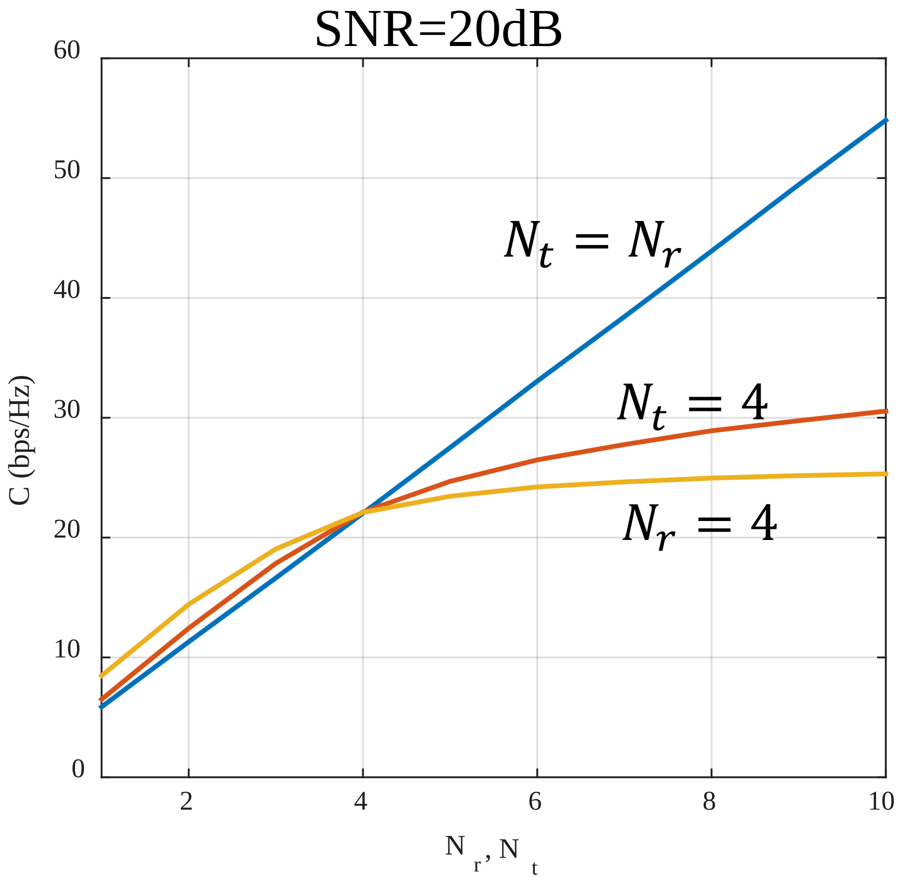
4×4 MIMO





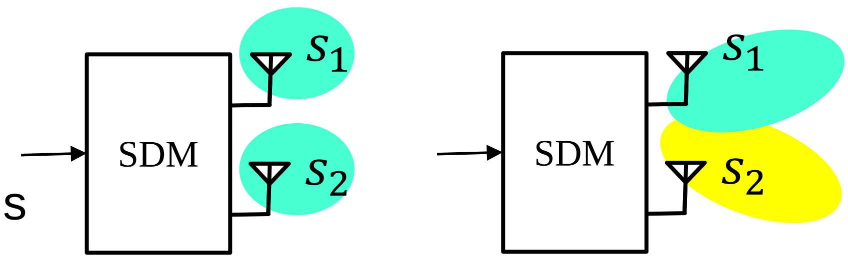
MIMO Channel Capacity (4)

- The ergodic capacity with i.i.d. (independent identical distributed) complex Gaussian channel gains



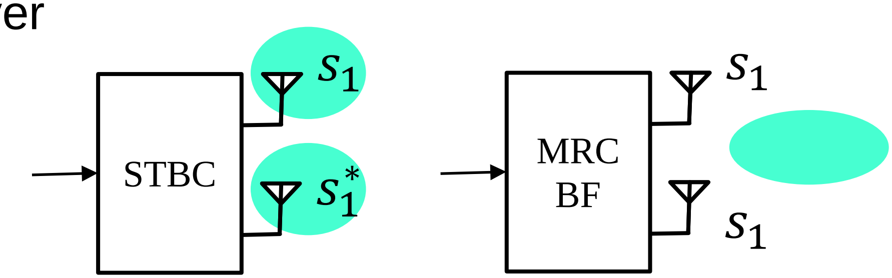
MIMO Operations

- Spatial division multiplexing (SDM)
  - The different signals are sent over each transmit antenna
  - Channel rank ≥ 2 → multiple streams
    - w/o CSIT: SDM
    - w/ CSIT: Eigenmode-SDM (E-SDM)



SDM E-SDM

- Diversity or beamforming
  - The same signal weighted is sent over each transmit antenna
  - Channel rank = 1 → single stream
    - w/o CSIT: Space-time block coding (STBC)
    - w/ CSIT: MRC



STBC MRC

Spatial Filtering

- Receive algorithm for SDM

$$\hat{\boldsymbol{s}} = \boldsymbol{W}^T\boldsymbol{y} = \boldsymbol{W}^T\boldsymbol{H}\boldsymbol{x} + \boldsymbol{W}^T\boldsymbol{n}$$

- Zero Forcing (ZF)

$$\boldsymbol{W} = \boldsymbol{H}^+ = (\boldsymbol{H}^H\boldsymbol{H})^{-\mathbf{1}}\boldsymbol{H}^H$$

- Minimum mean square error (MMSE)

$$\boldsymbol{W}^T = \left(\boldsymbol{H}^H\boldsymbol{H} + \frac{N_t}{\gamma_0}\boldsymbol{I}_{N_t}\right)^{-\mathbf{1}}\boldsymbol{H}^H$$

- Maximum likelihood detection (MLD)

$$\min_{\tilde{\boldsymbol{s}}_q}\left\|\boldsymbol{y} - \boldsymbol{H}\tilde{\boldsymbol{s}}_q\right\|^2$$

Spatial Filtering

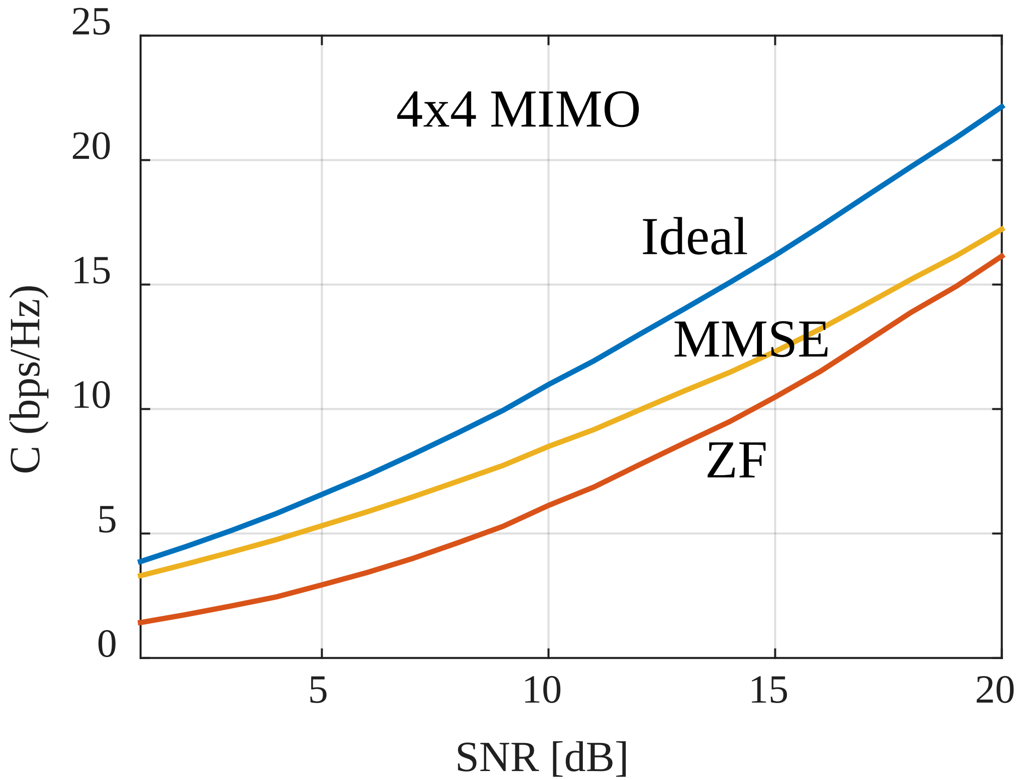
- Capacity

ZF

$$C = E\left[\sum_{i=1}^{N_t} \log\left(\frac{\gamma_0}{N_t \boldsymbol{w}_i^H \boldsymbol{w}_i} + 1\right)\right]$$

MMSE

$$C = -E\left[\sum_{i=1}^{N_t} \log\left(\frac{N_t}{\gamma_0}\left(\boldsymbol{H}^H \boldsymbol{H} + \frac{N_t}{\gamma_0} \boldsymbol{I}_{N_t}\right)^{-1}\right)\right]$$



SDM (pp.18)

$$C = E\left[\sum_{i=1}^{N_{\min}} \log\left(\frac{\sigma_i^2}{N_t}\gamma_0 + 1\right)\right]$$