電子情報通信特論
Advanced Course of Electronics, Information and Communications

Diversity

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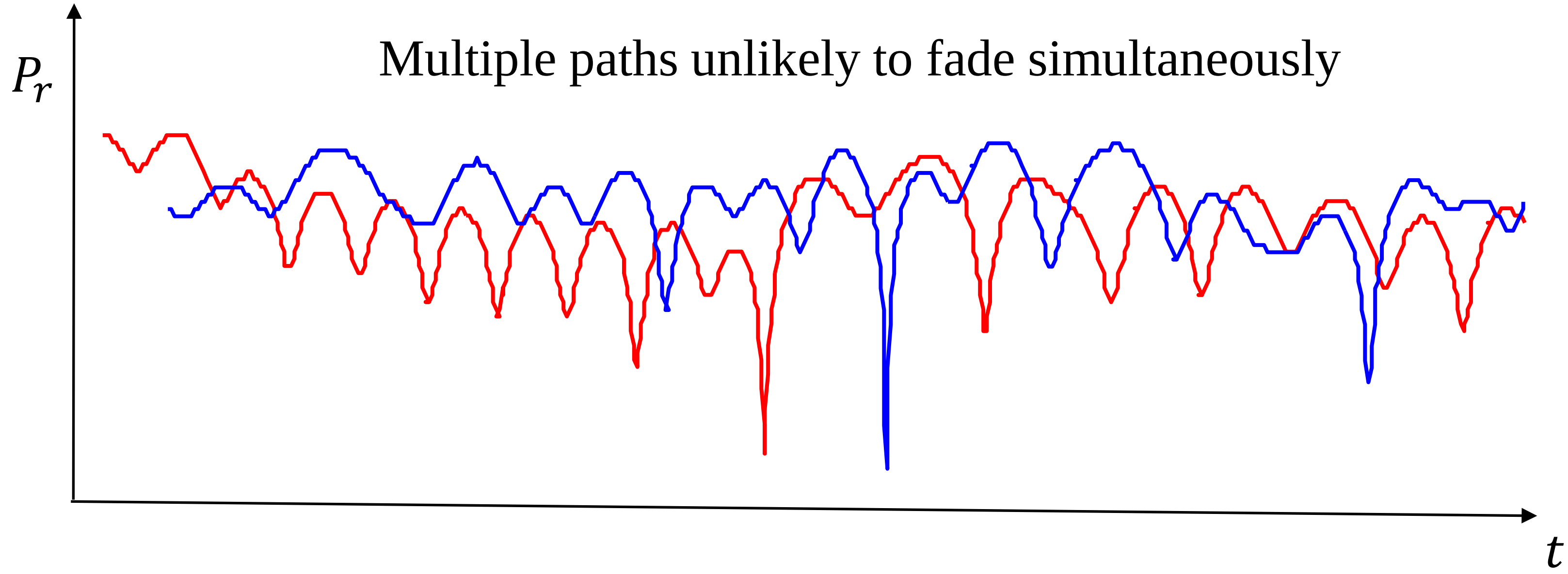
NiiGATA UNiVERSiTY

Outline

- Various diversity schemes
- Receiver diversity
  - Selection combining
  - Maximum ratio combining
- Transmitter diversity
  - With CSIT
  - Without CSIT (Alamouti’s scheme)

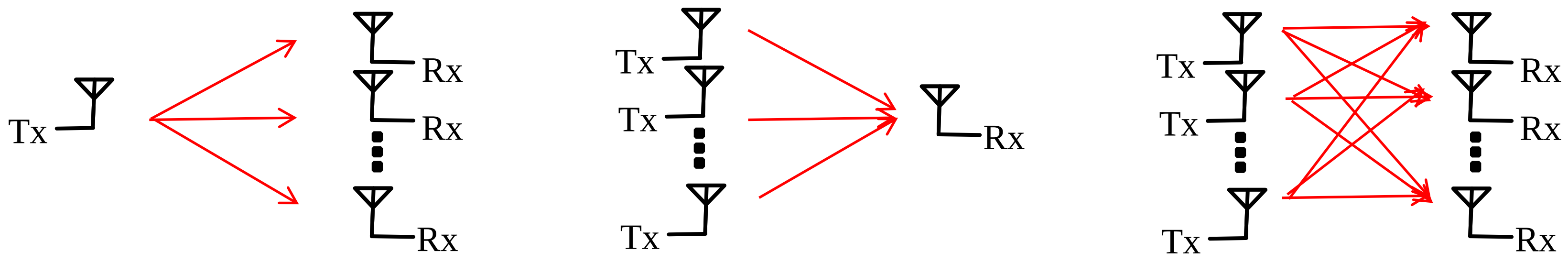
What Is Diversity Technique ?

- Basic Idea
  - Send same bits over independent fading paths
    - Independent fading paths obtained by time, space, frequency, or polarization diversity
  - Combine paths to mitigate fading effects



Space Diversity

- Using multiple antennas at Rx or Tx or both



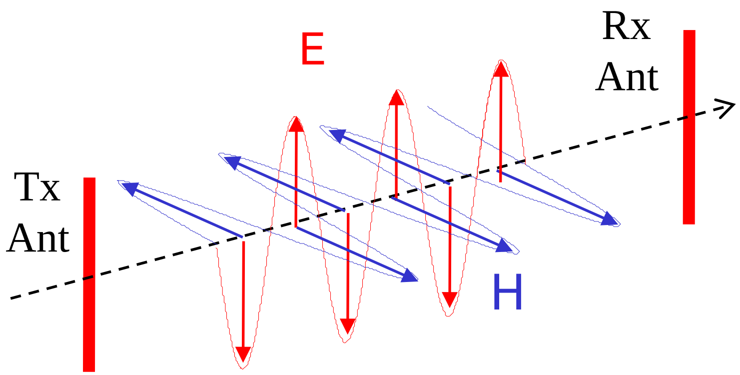
SIMO Diversity MISO Diversity MIMO Diversity

Antenna separation > half wavelength

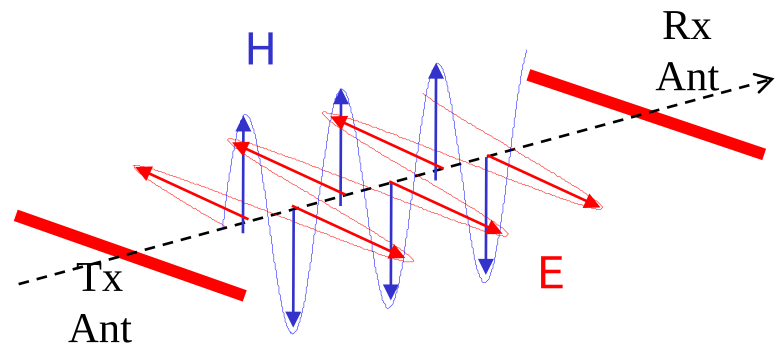
- Array gain & Diversity gain
  - Array gain is from noise averaging (AWGN and fading)
  - Diversity gain is change in BER slope (fading)

Polarization Diversity

- Using antenna's polarization at Rx or Tx or both



Vertical Polarization

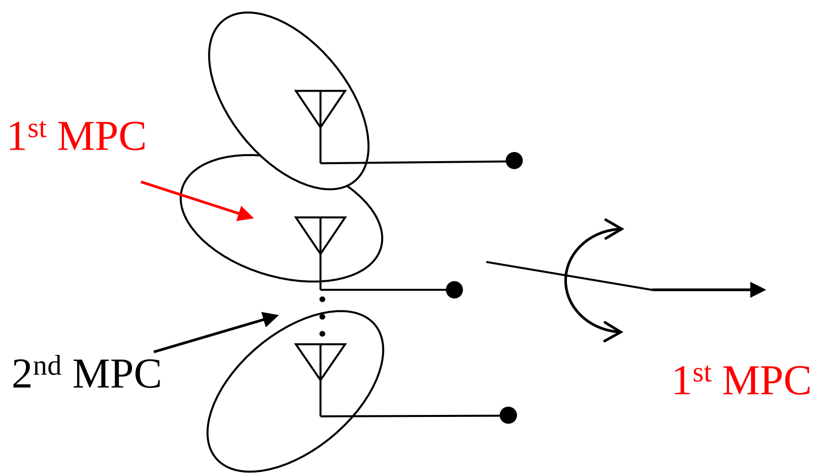


Horizontal Polarization

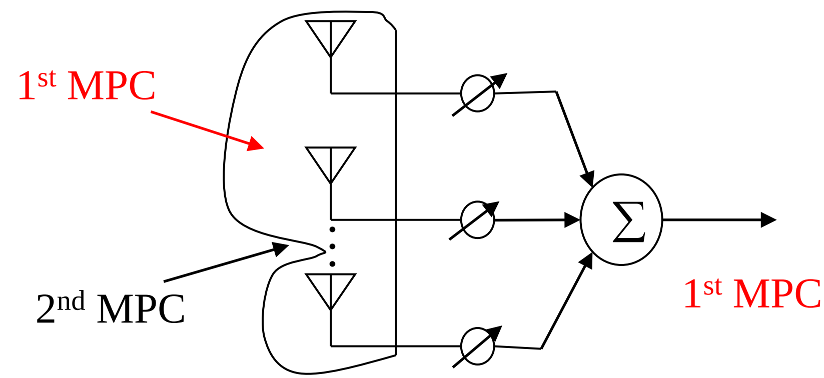
- Features
  - Two diversity branches are available at the same position (save space)
  - Only two diversity branches are available
  - Polarization diversity loss (3dB) because Tx/Rx power is divided between two different polarized antennas

Angle (directional) Diversity

- Multiple antennas with narrow beam width
  - There is no fading
  - Large number of antennas with sufficient gain are necessary
- Smart antennas



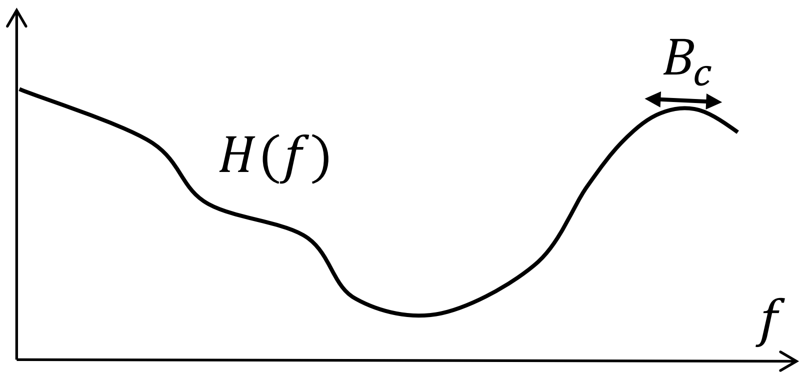
Multiple directional antennas

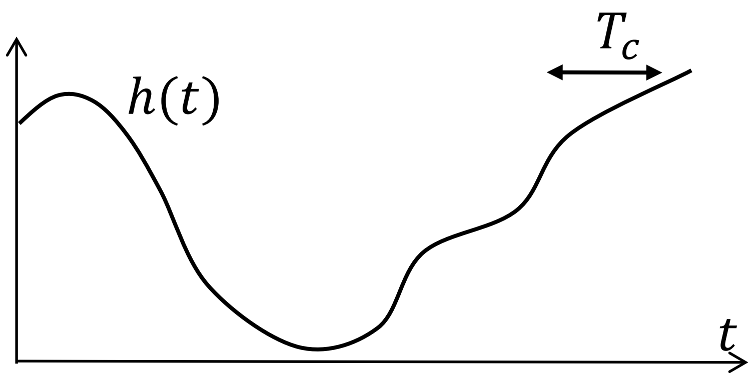


Smart antennas (digital signal processing)

Frequency and Time Diversity

- Frequency diversity
  - Transmitting the same narrowband signal at different frequencies
  - Frequency difference should be larger than coherence bandwidth
- Time diversity
  - Transmitting the same signal at different times
  - Times difference should be larger than coherence time





Receiver Diversity

- Selection Combining
  - Fading path with highest gain used
- Maximal Ratio Combining
  - All paths co-phased and summed with optimal weighting to maximize combiner output SNR
- Equal Gain Combining
  - All paths co-phased and summed with equal weighting

System Model

- M-branch diversity system
  - Complex weight for coherent combining: $\alpha_i = a_i e^{-\mathrm{j}\theta_i}$
  - Combined signal

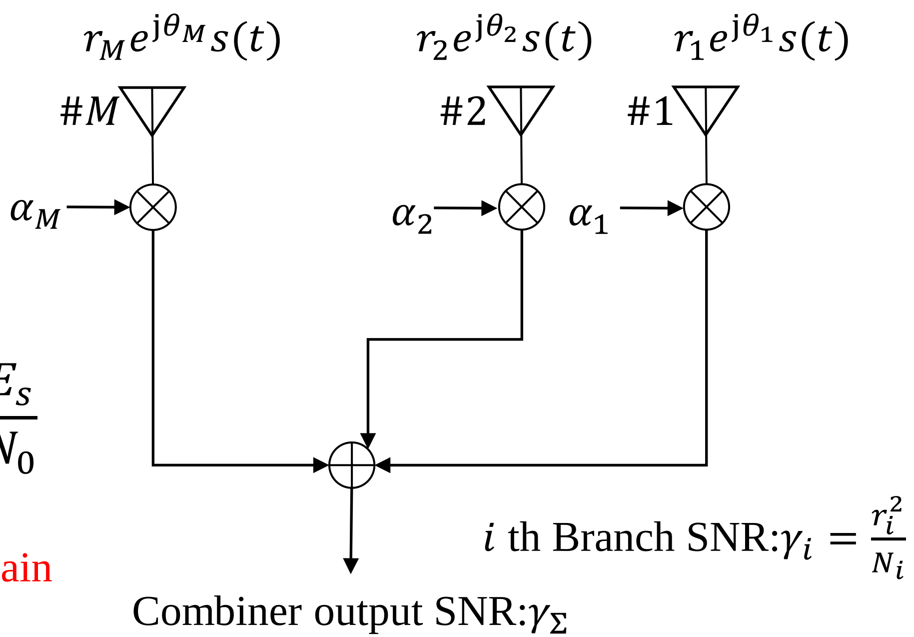
$$\alpha_\Sigma s(t) = \left(\sum r_i a_i\right) s(t)$$

- SNR for AWGN when $\gamma_i = \frac{E_s}{N_0}$ Same SNR

$$\gamma_\Sigma = \frac{\left(\sum_{i=1}^{M} a_i r_i\right)^2}{N_0 \sum_{i=1}^{M} a_i^2} = \frac{\left(\sum_{i=1}^{M} \frac{E_s}{\sqrt{N_0}}\right)^2}{N_0 \sum_{i=1}^{M} \frac{E_s}{N_0}} = \boxed{M} \frac{E_s}{N_0}$$

where $a_i = \frac{r_i}{\sqrt{N_0}}$

Array gain



Remind! $\mathrm{SNR} = \frac{P_s}{P_N} = \frac{E_s}{\frac{N_0}{2} 2BT_s} = \frac{E_s}{N_0}$ (when $BT_s = 1$)

Diversity Gain & Diversity Order

- Array Gain
  - $A_g = \frac{\bar{\gamma}_\Sigma}{\bar{\gamma}}$

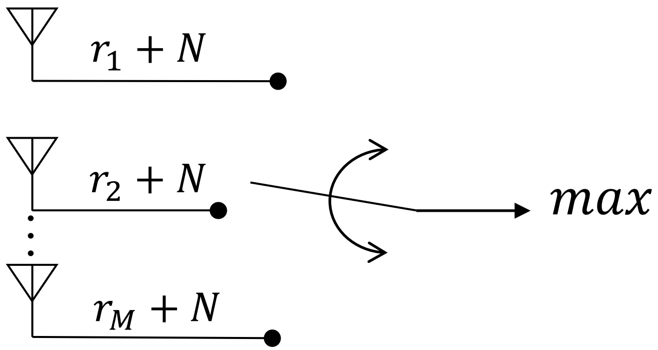
    $\bar{\gamma}_\Sigma$: combiner output SNR
    $\bar{\gamma}$: SNR at a single-antenna element
  - Averaging over the noise at different antennas
    → $\bar{\gamma}_\Sigma$ is larger than $\bar{\gamma}$

- Diversity Gain & Diversity Order
  - Diversity gain: performance advantage of $\bar{P}_s$ and P_{out}
  - $\bar{P}_s = \int_0^\infty P_s(\gamma) p_{\gamma_\Sigma}(\gamma)\, d\gamma = c\bar{\gamma}^{-\boxed{M}}$

    Diversity order
  - $P_{\text{out}} = \int_0^{\gamma_0} p_{\gamma_\Sigma}(\gamma) d\gamma$

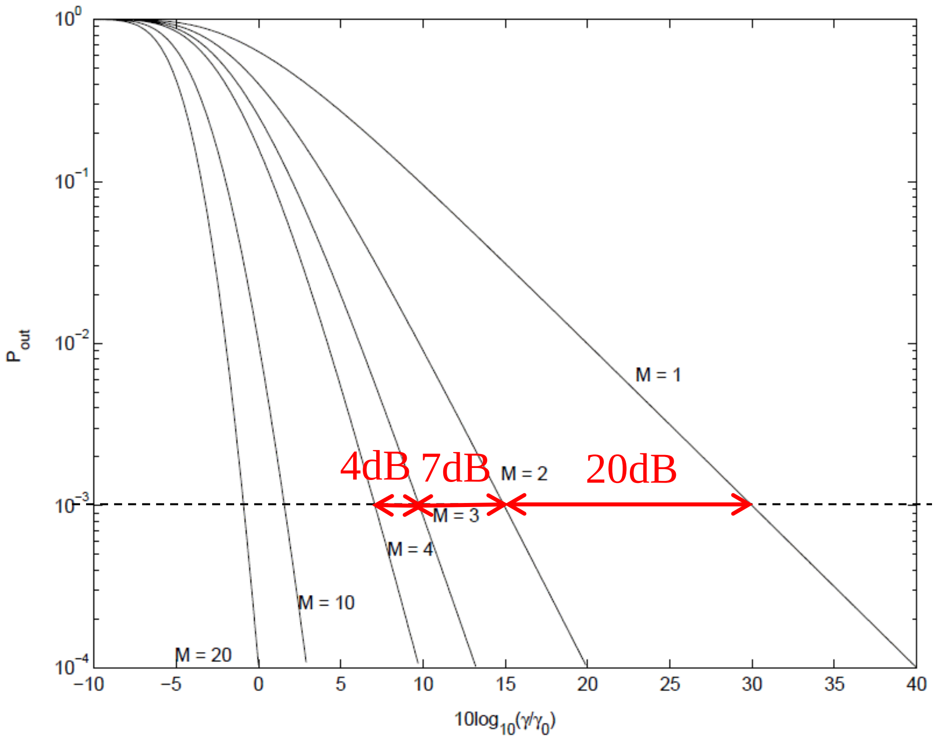
Selection Combining

- Combiner SNR is the maximum of the branch SNRs.
- Just selecting maximum → Co-phasing is not necessary
- $P_{\gamma_\Sigma}(\gamma) = p(\gamma_\Sigma < \gamma) = p(\max[\gamma_1, \gamma_2, \dots, \gamma_M] < \gamma) = \prod_{i=1}^{M} p(\gamma_i < \gamma)$
- Rayleigh fading
  - $P_{\text{out}}(\gamma_0) = P_{\gamma_\Sigma}(\gamma_0) = \prod_{i=1}^{M} \left[1 - e^{-\gamma_0/\bar{\gamma}_i}\right] = \left(1 - e^{-\gamma_0/\bar{\gamma}}\right)^M$
  - $p_{\gamma_\Sigma}(\gamma) = M/\bar{\gamma}\left(1 - e^{-\gamma_0/\bar{\gamma}}\right)^{M-1} e^{-\gamma_0/\bar{\gamma}}$
  - $\bar{\gamma}_\Sigma = \int_0^\infty \gamma p_{\gamma_\Sigma}(\gamma)\, d\gamma = \bar{\gamma} \sum_{i=1}^{M} i^{-1}$

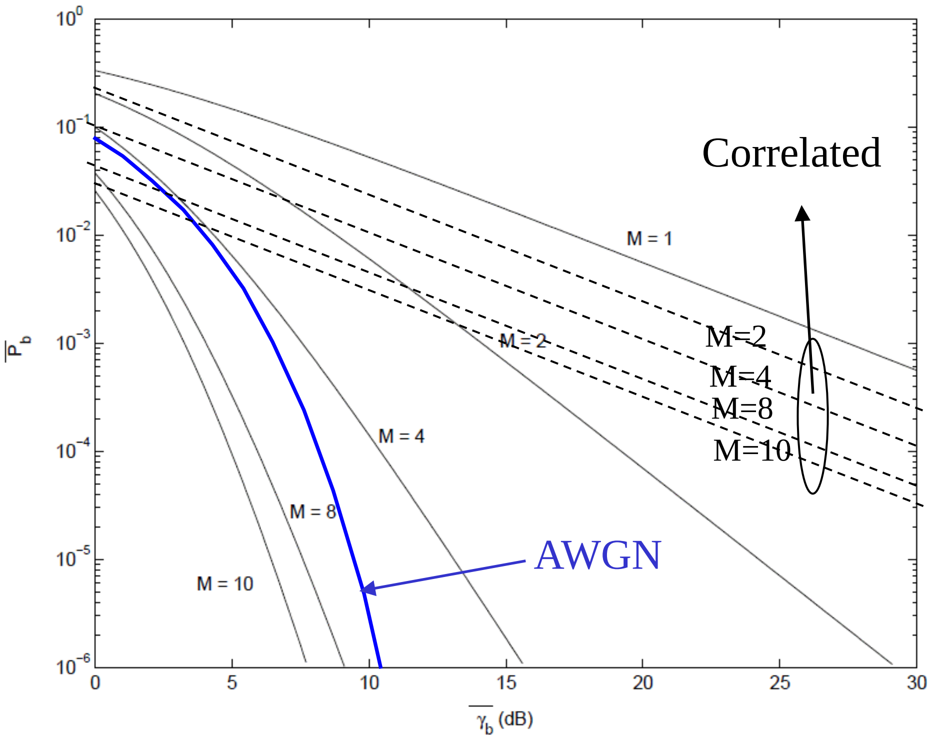


Selection Combining

- SC with M branch (i.i.d. Rayleigh fading channel)



Outage prob.



Ave. BER for BPSK

Maximum Ratio Combining

- Weighted sum of all branches coherently
- Output SNR of combiner

$$\gamma_\Sigma = \frac{r^2}{N_{tot}} = \frac{1}{N_0}\frac{\left(\sum_{i=1}^{M} a_i r_i\right)^2}{\sum_{i=1}^{M} a_i^2}$$

where the envelope of the combiner output $r = \sum_{i=1}^{M} a_i r_i$

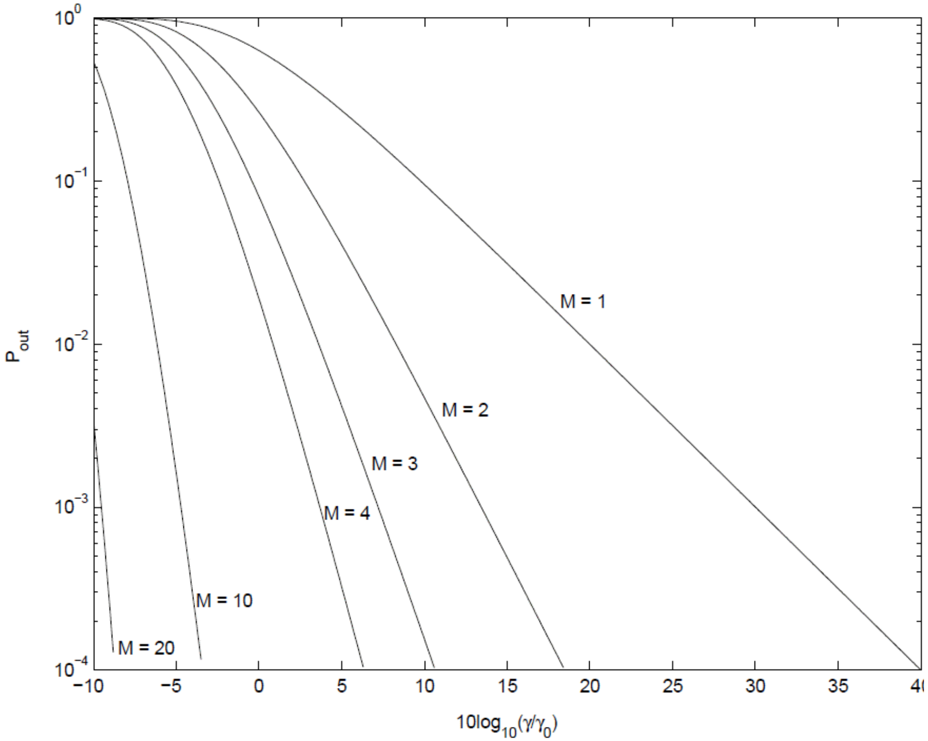
By Cauchy-Schwarz inequality $\left(\sum_{i=1}^{M} a_i r_i\right)^2 \le \left(\sum_{i=1}^{M} a_i^2\right)\left(\sum_{i=1}^{M} r_i^2\right)$

$$\gamma_\Sigma \le \frac{1}{N_0}\frac{\left(\sum_{i=1}^{M} a_i^2\right)\left(\sum_{i=1}^{M} r_i^2\right)}{\sum_{i=1}^{M} a_i^2} = \sum_{i=1}^{M}\frac{r_i^2}{N_0} = \sum_{i=1}^{M}\gamma_i$$

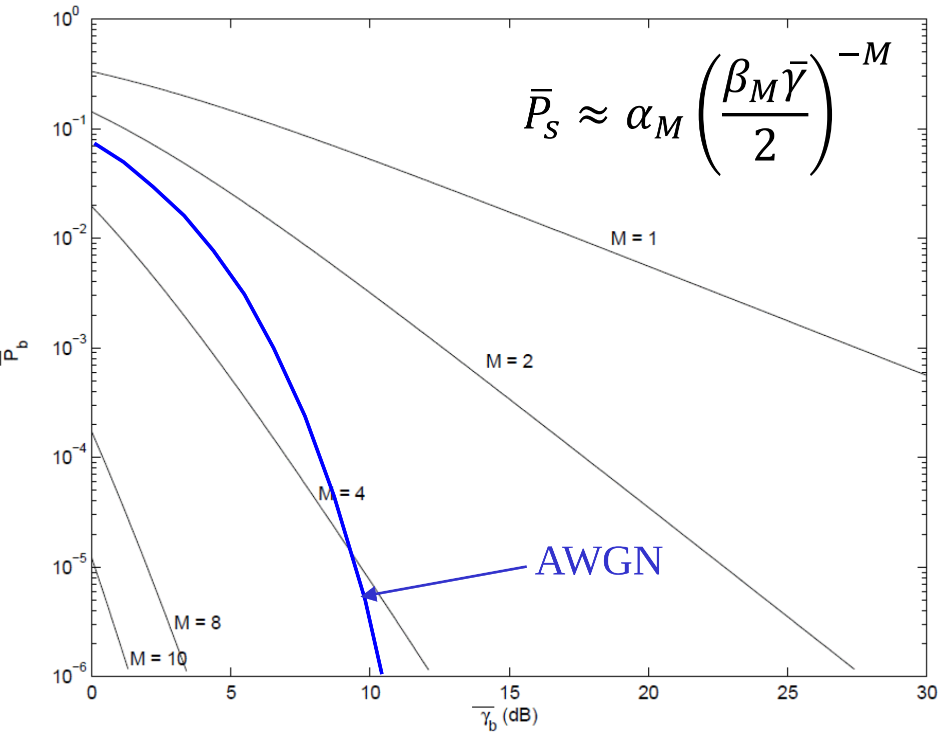
where the equality holds when $a_i = c r_i$, for example, $a_i = \frac{r_i}{\sqrt{N_0}}$

Maximum Ratio Combining

- MRC with M branches (i.i.d. Rayleigh fading channel)



Outage prob.

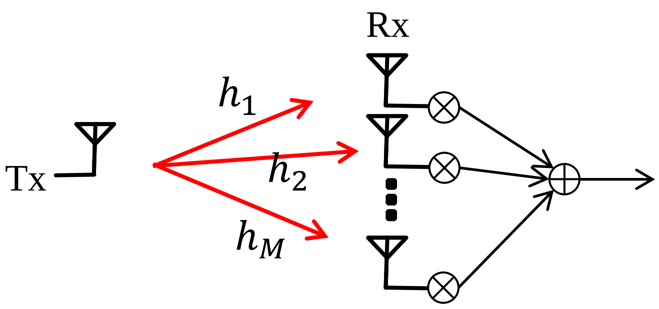


Ave. BER for BPSK

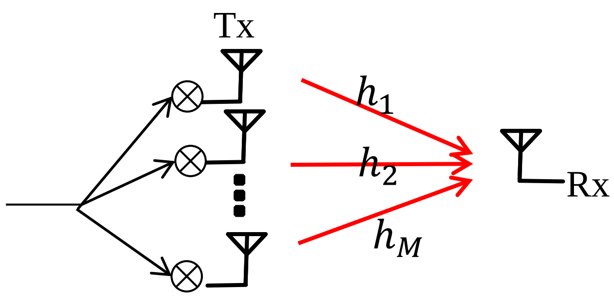
Transmitter Diversity

- Remember co-phasing is necessary for combiner
- Two cases are considered
  - Channel Known at Transmitter
    Opposite way (MRC) to the receiver diversity is possible by pre-coding at each transmitter antennas
  - Channel Unknown at Transmitter

    → Alamouti's scheme



Receiver Diversity

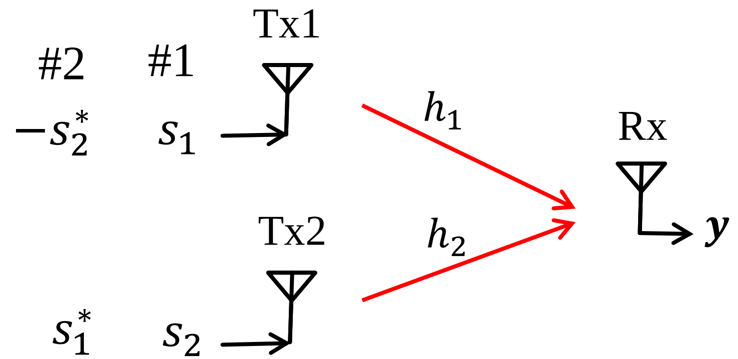


Transmitter Diversity

Alamouti's Tx Diversity

- No CSIT; channel is unknown at transmitter
- Space and time diversity with two Tx antennas
- Transmit two symbols simultaneously twice as

$$\boldsymbol{y} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \boldsymbol{H}_A \boldsymbol{s} + \boldsymbol{n}$$



$$\boldsymbol{z} = \boldsymbol{H}_A^H \boldsymbol{y} = \boldsymbol{H}_A^H \boldsymbol{H}_A \boldsymbol{s} + \boldsymbol{H}_A^H \boldsymbol{n}$$

$$= (|h_1|^2 + |h_2|^2) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{s} + \boldsymbol{H}_A^H \boldsymbol{n}$$

$$= \begin{bmatrix} (|h_1|^2 + |h_2|^2) s_1 + \tilde{n}_1 \\ (|h_1|^2 + |h_2|^2) s_2 + \tilde{n}_2 \end{bmatrix}$$

$$\gamma_1 = \frac{(|h_1|^2 + |h_2|^2) E_s}{2N_0}$$

$$\boldsymbol{E}[s_1 s_1^*] = E_s/2$$
$$\boldsymbol{E}[\tilde{n}_1 \tilde{n}_1^*] = (|h_1|^2 + |h_2|^2) N_0$$