電子情報通信特論
Advanced Course of Electronics, Information and Communications

Statistical Multipath Channel Models

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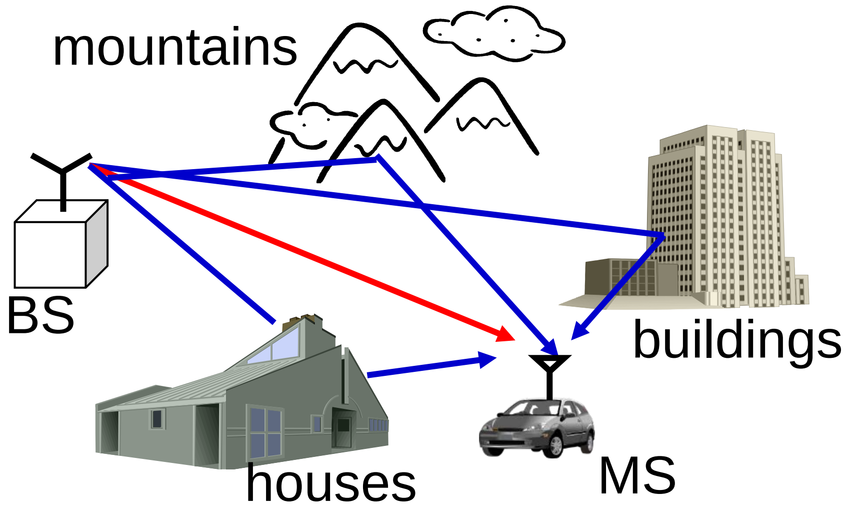
Outline

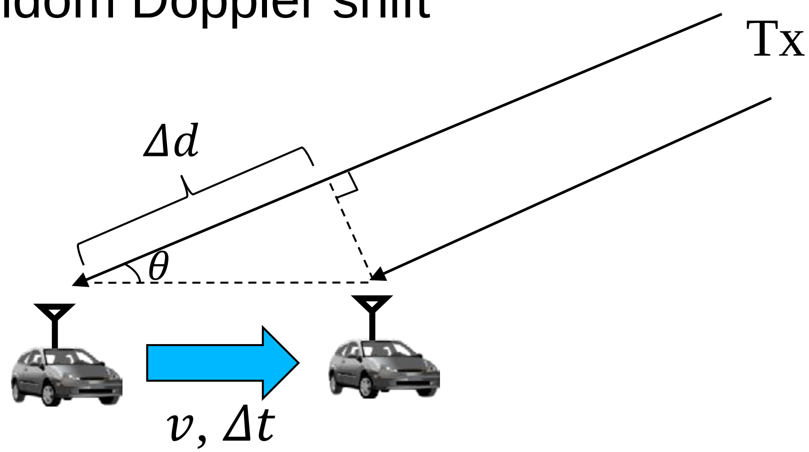
- Statistical Multipath Channel Model
  - Mathematical Representation
  - Narrowband Fading Model
  - Wideband Fading Model



Multipath Environment

Statistical Multipath Channel

- Random # of multipath components, each with
  - Random amplitude and phase
  - Random delay
  - Random Doppler shift



$$\Delta d = v\Delta t\cos\theta$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta d = \frac{2\pi}{\lambda}v\Delta t\cos\theta$$

$$f_D = \frac{\Delta\phi}{2\pi}\frac{1}{\Delta t} = \frac{v\cos\theta}{\lambda}$$

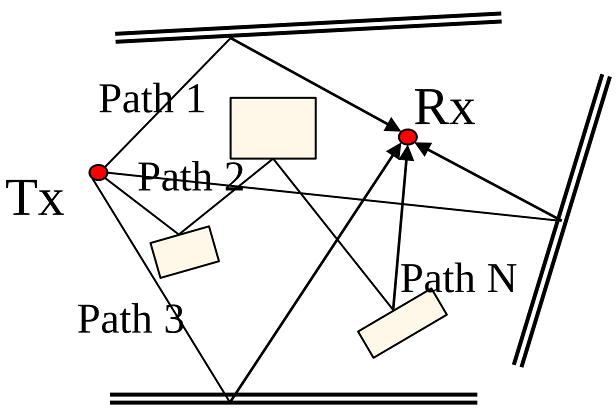
- Random components change with time
  ⇒ Time-varying channel impulse response

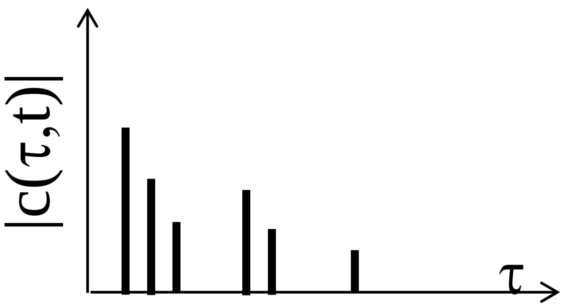
Time Varying Channel Impulse Response

- Response of channel at t to impulse at $t-\tau$:

$$c(\tau,t)=\sum_{n=1}^{N(t)}\alpha_n(t)e^{-j\varphi_n(t)}\delta\big(\tau-\tau_n(t)\big)$$

  - t is time when impulse response is observed
  - $t-\tau$ is time when impulse put into the channel
  - τ is how long ago impulse was put into the channel for the current observation
    - path delay for multipath component currently observed





Channel impulse response

Received Signal

- Signal representation

Bandpass $s(t) \longrightarrow \boxed{h(\tau, t)} \longrightarrow r(t)$

Lowpass $u(t) \longrightarrow \boxed{c(\tau, t)} \longrightarrow v(t)$

$\alpha_n(t)$ $\tau_n(t)$ $\phi_{D_n}(t)$

Random Amp. Random Delay Random Doppler shift

$$s(t) = A\cos\big(2\pi f_c t + \theta(t)\big) = \mathrm{Re}\big[u(t)e^{\mathrm{j}2\pi f_c t}\big]$$

$$r(t) = \mathrm{Re}\left[\sum_{n=1}^{N(t)} \alpha_n(t) u\big(t - \tau_n(t)\big) e^{\mathrm{j}\left(2\pi f_c\left(t-\tau_n(t)\right)+\phi_{D_n}(t)\right)}\right]$$

Received Signal

- Received signal

$$r(t) = \mathrm{Re}\left[\sum_{n=1}^{N(t)} \alpha_n(t) u\big(t - \tau_n(t)\big) e^{\mathrm{j}\left(2\pi f_c(t-\tau_n(t)) + \phi_{D_n}(t)\right)}\right]$$
$$= \mathrm{Re}\left[\int_{-\infty}^{\infty} c(\tau,t) u(t-\tau) d\tau \cdot e^{\mathrm{j}2\pi f_c t}\right]$$

$$\boxed{c(\tau,t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-\mathrm{j}\varphi_n(t)} \delta\big(\tau - \tau_n(t)\big)}$$

where $\varphi_n(t) = \underbrace{2\pi f_c \tau_n(t)}_{\text{delay}} - \underbrace{\phi_{D_n}(t)}_{\text{Doppler}}$

- Equivalent baseband representation

$$v(t) = \int_{-\infty}^{\infty} \underline{c(\tau,t)} u(t-\tau) d\tau$$

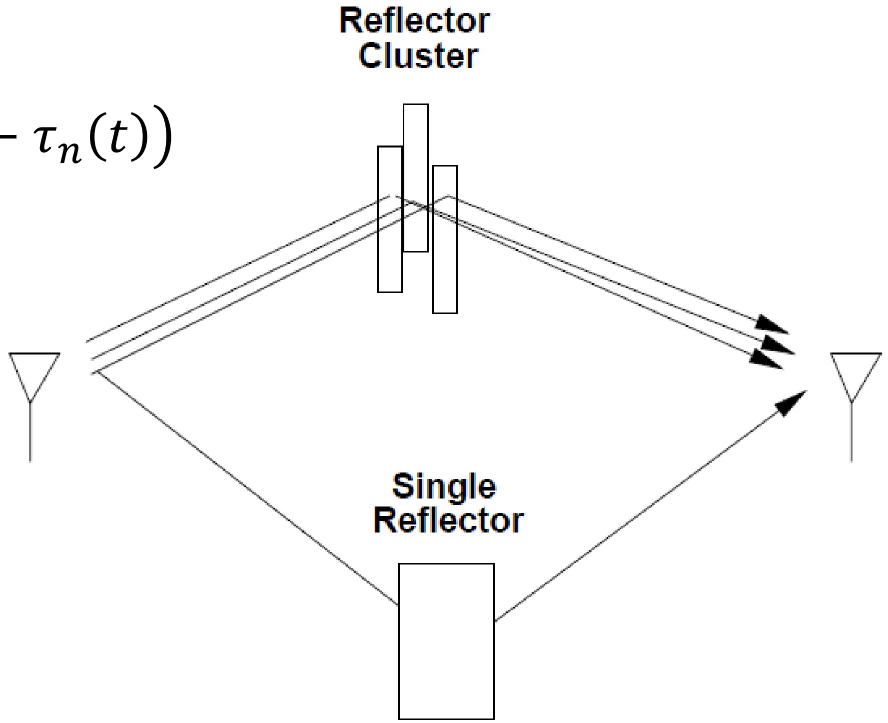
遅延広がりが時間的に変化

Multipath Components (MPCs)

- Multipath due to single reflector
- Multipath due to multiple reflectors (reflector cluster)

  →Generated MPC with similar delays

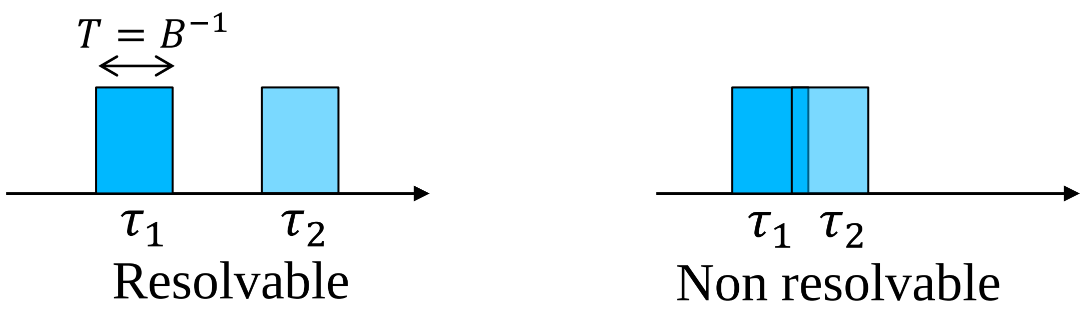
$$c(\tau,t)=\sum_{n=1}^{N(t)}\alpha_n(t)e^{-\mathrm{j}\varphi_n(t)}\delta\big(\tau-\tau_n(t)\big)$$



Multipath Components

- Resolvable multipath components
  - Delay difference significantly exceeds the inverse signal bandwidth
  - Delay between multipath components are larger than symbol duration

$$|\tau_2 - \tau_1| \gg B^{-1}$$



- Non-resolvable components
  - Typically combined into a single multipath component with amplitude and phase corresponding to sum of different components → undergo fast fading (Narrowband fading)

Characterization of IR

- Characterized as random processes

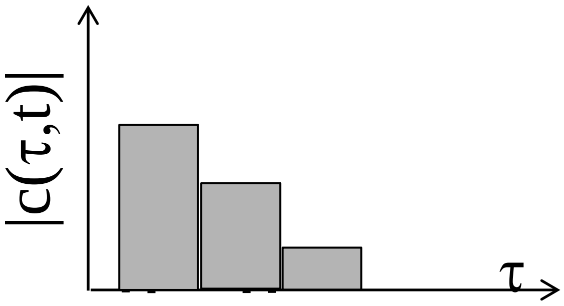
$$c(\tau, t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\varphi_n(t)} \delta\big(\tau - \tau_n(t)\big)$$

$$\underset{\substack{\uparrow \\ \text{Random} \\ \text{amp.}}}{\alpha_n(t)} \quad \underset{\substack{\uparrow \\ \text{Random} \\ \text{Delay}}}{\tau_n(t)} \quad \underset{\substack{\uparrow \\ \text{Random} \\ \text{Doppler shift}}}{\phi_{D_n}(t)}$$

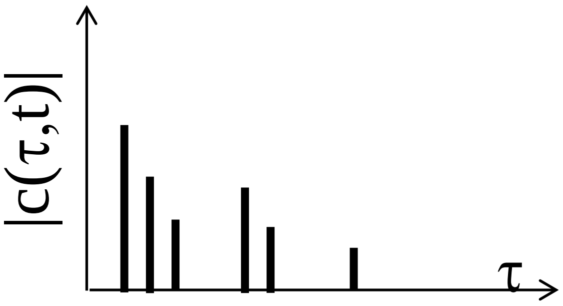
- Wide sense stationary (WSS):
  - Mean and variance of a random process does not change over time (Probability distribution is same at all times)
- Ergodic:
  - Statistical properties (mean and variance) can be deduced from sufficiently long sample (realization) of the process.

Narrowband or Wideband Fading

- Narrowband fading
  - Delay spread is small relative to inverse of signal bandwidth
  - LOS and MPCs arrive at relatively the same time → Non resolvable received signals
  - Rapid fluctuation (fast fading)
- Wideband fading
  - Delay spread is large relative to inverse of signal bandwidth
  - MPCs are resolvable into discrete components



Narrowband IR

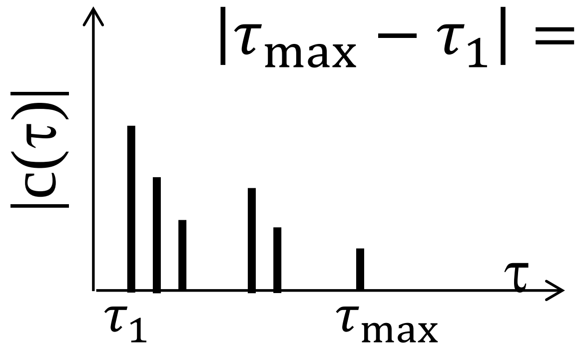


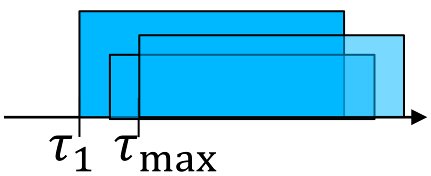
Wideband IR

Narrowband Channel

- Consider a system with delay spread τ_{m} which is small relative to the inverse of the bandwidth of the transmitted signal

$$|\tau_{\max} - \tau_1| = \tau_{\mathrm{m}} \ll B^{-1}$$





Non resolvable

- MPCs arrive at approximately the same time, namely $u(t - \tau_i) \approx u(t)$

Narrowband Channel

- Assuming $u(t-\tau_i) \approx u(t)$,

$$r(t) = \mathrm{Re}\left[\underbrace{u(t)e^{\mathrm{j}2\pi f_c t}}_{\text{Modulated carrier}} \underbrace{\left(\sum_{n=1}^{N(t)} \alpha_n(t) e^{-\mathrm{j}\varphi_n(t)}\right)}_{\text{Random scale factor}}\right]$$

- To characterize the random scale factor, a transmitted signal $s(t)$ is chosen as

$$s(t) = \mathrm{Re}\left[\underbrace{u \cdot e^{\mathrm{j}2\pi f_c t}}_{\text{Unmodulated carrier}}\right] = \cos(2\pi f_c t + \phi_0)$$

Continuous wave (CW)

Narrowband Channel

- Received signal

$$r(t) = \mathrm{Re}\left[\underbrace{\boxed{\left(\sum_{n=1}^{N(t)} \alpha_n(t) e^{-\mathrm{j}\varphi_n(t)}\right)}}_{\text{Random scale factor}} e^{\mathrm{j}2\pi f_c t}\right] = r_I(t)\cos 2\pi f_c t - r_Q(t)\sin 2\pi f_c t$$

$$\left.\begin{aligned} r_I(t) &= \sum_{n=1}^{N(t)} \alpha_n(t)\cos\varphi_n(t) \\ r_Q(t) &= -\sum_{n=1}^{N(t)} \alpha_n(t)\sin\varphi_n(t) \end{aligned}\right\}$$

Approaches Gaussian random process with large $N(t)$ (CLT; central limit theorem)

$$\varphi_n(t) = 2\pi f_c \tau_n(t) - \phi_{D_n} - \phi_0$$

Statistical Property

- Assume that there is no dominant LoS component
- Slowly time varying

$$\alpha_n(t) \approx \alpha_n, \tau_n(t) \approx \tau_n, f_{D_n}(t) \approx f_{D_n}$$

- Phase of n th multipath component becomes:

$$\varphi_n(t) = \underbrace{2\pi f_c \tau_n}_{\text{dominantly change}} - \phi_{D_n}(t) - \phi_0$$

Uniformly distributed $[-\pi \quad \pi]$

$\because \phi_{D_n}(t) = 2\pi f_{D_n} t$

Doppler shift

Statistical Property

- Mean

$$E[r_I(t)] = E\left[\sum_n \alpha_n \cos\varphi_n(t)\right] = \sum_n E[\alpha_n]E[\cos\varphi_n(t)] = 0$$

$\because$ α_n and φ_n are independent

$$E\left[r_Q(t)\right] = E[r(t)] = 0$$

→ zero mean Gaussian process

- Cross correlation

$$E\left[r_I(t)r_Q(t)\right] = E\left[\sum_n \alpha_n \cos\varphi_n(t) \sum_m \alpha_m \sin\varphi_m(t)\right]$$

$$= \sum_n \sum_m E[\underline{\alpha_n \alpha_m}]E[\cos\varphi_n(t) \sin\varphi_m(t)]$$

independent $(m \neq n)$

$$= \sum_n \sum_m E[\alpha_n^2]E[\cos\varphi_n(t) \sin\varphi_m(t)] = 0$$

Statistical Property

- Autocorrelation for $r_I(t)$

$$
\begin{aligned}
A_{r_I}(t, t+\tau) &= E[r_I(t) r_I(t+\tau)] \\
&= \sum_n E[\alpha_n^2] E[\cos\varphi_n(t) \cos\varphi_n(t+\tau)] \\
&= 0.5 \sum_n E[\alpha_n^2] \{ E[\cos 2\pi f_{D_n}\tau] + \\
&\qquad E[\cos(4\pi f_c \tau_n - 4\pi f_{D_n} t - 2\pi f_{D_n}\tau - 2\phi_0)] \} \\
&= 0.5 \sum_n E[\alpha_n^2] E[\cos 2\pi f_{D_n}\tau] \\
&= 0.5 \sum_n E[\alpha_n^2] E\left[\cos\left(\frac{2\pi v\tau}{\lambda}\cos\theta_n\right)\right] \quad \text{assuming } f_{D_n} \text{ is fixed}
\end{aligned}
$$

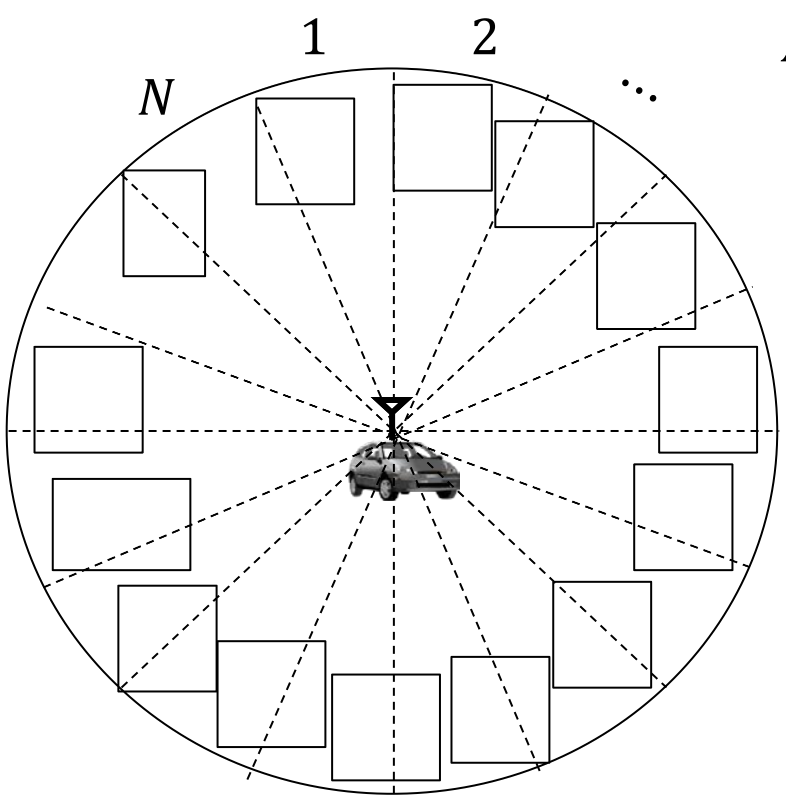
  - $A_{r_I}(\tau) = A_{r_Q}(\tau)$ → Wide sense stationary (WSS)
    広義定常性

- Autocorrelation for $r(t)$

$$
A_r(\tau) = A_{r_I}(\tau) \cos 2\pi f_c \tau - A_{r_I r_Q}(\tau) \sin 2\pi f_c \tau
$$

Clarke's Model

- Uniform scattering model (further developed by Jake)



$$\Delta\theta = 2\pi/N$$

$$A_{r_I}(\tau) = 0.5\sum_n E[\alpha_n^2]E\left[\cos\left(\frac{2\pi v\tau}{\lambda}\cos\theta_n\right)\right]$$
$$= \frac{P_r}{N}\sum_n \cos\left(\frac{2\pi v\tau}{\lambda}\cos n\Delta\theta\right)$$

where $P_r = A_r(0) = 0.5N \cdot E[\alpha_n^2]$

$$A_{r_I}(\tau) = \frac{P_r}{2\pi}\sum_n \cos\left(\frac{2\pi v\tau}{\lambda}\cos n\Delta\theta\right)\Delta\theta$$
$$\approx \frac{P_r}{2\pi}\int_0^{2\pi} \cos\left(\frac{2\pi v\tau}{\lambda}\cos\theta\right)d\theta$$

when $N \to \infty$, and $\Delta\theta \to 0$

Clarke's Model

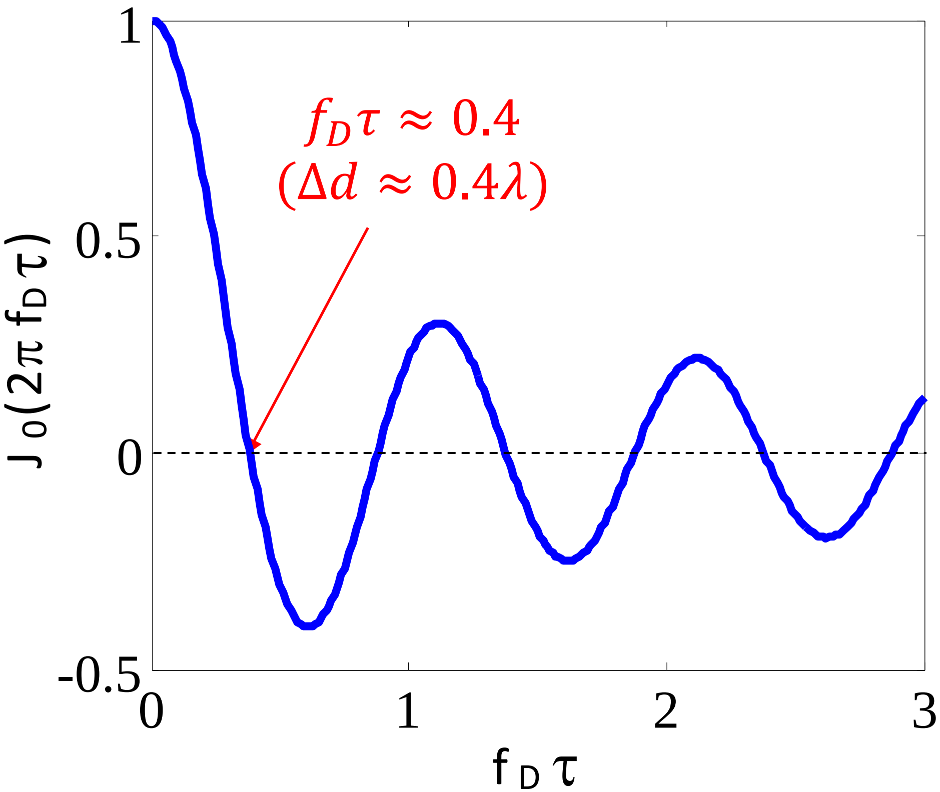
- Autocorrelation is expressed by Bessel function

v/λ: max Doppler

$$A_{r_I}(\tau) = \frac{P_r}{2\pi}\int_0^{2\pi} \cos\left(\frac{2\pi v \tau}{\lambda}\cos\theta\right) d\theta = P_r J_0(2\pi f_D \tau)$$

where $J_0(x) = \frac{1}{\pi}\int_0^{\pi} e^{-\mathrm{j}x\cos\theta} d\theta$

(the first kind, zeroth order)

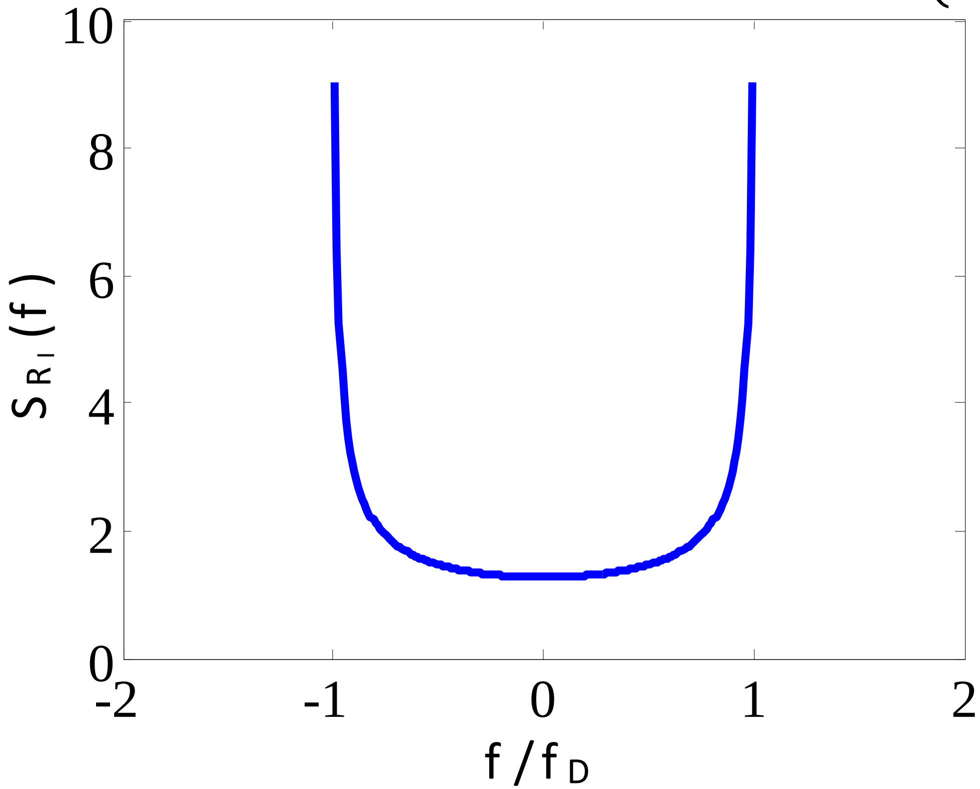


Decorrelates over approximately half wavelength
→ Ant. distance for diversity combining

Power Spectrum Density (PSD)

- By using Einstein-Wiener-Hinchin Theorem

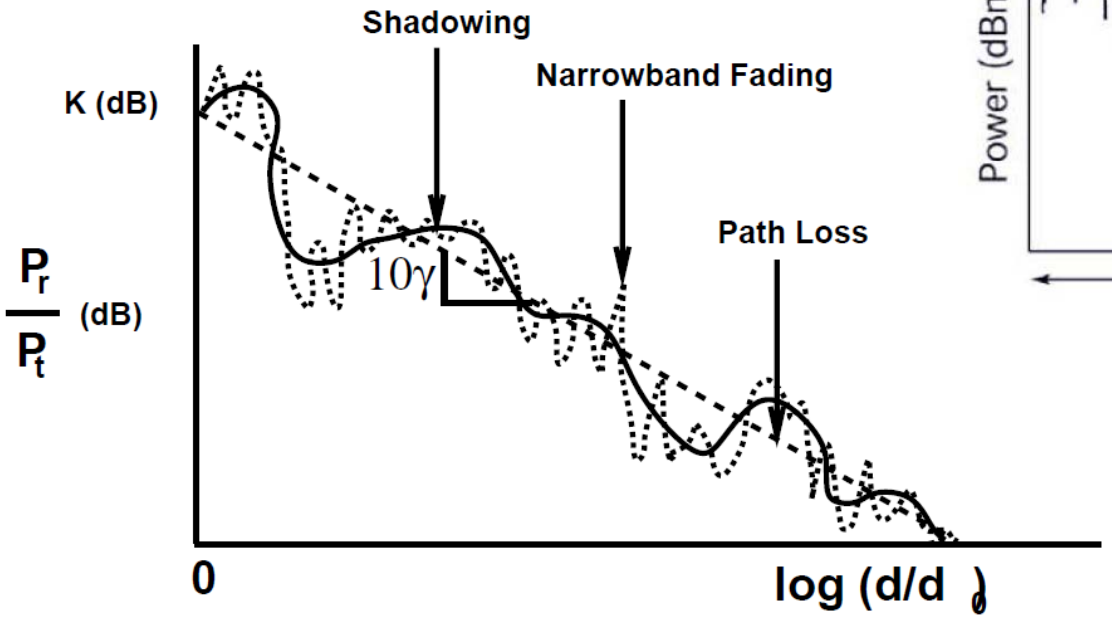
$$S_{r_I}(f) = S_{r_Q}(f) = \mathcal{F}\left[A_{r_I}(\tau)\right] = \begin{cases} \dfrac{P_r}{2\pi f_D} \dfrac{1}{\sqrt{1-(f/f_D)^2}} & |f| \leq f_D \\ 0 & \text{else} \end{cases}$$

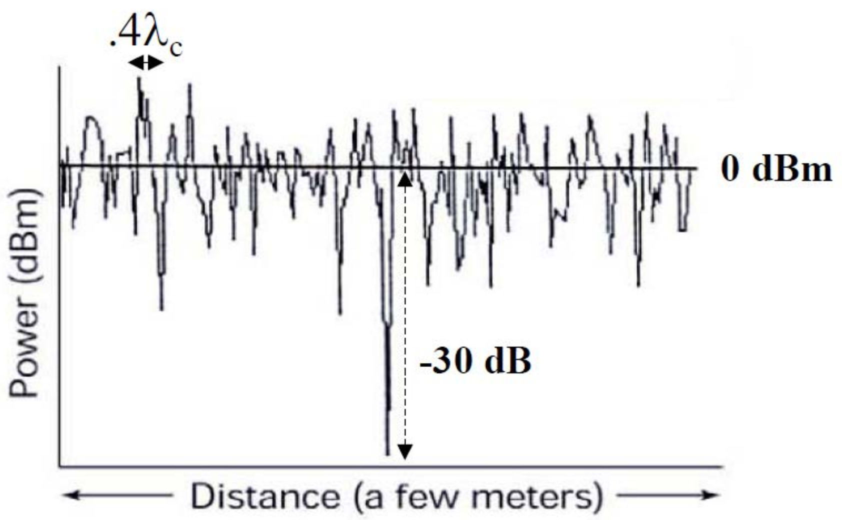


$$S_r(f) = \begin{cases} \dfrac{P_r}{4\pi f_D} \dfrac{1}{\sqrt{1-(|f-f_c|/f_D)^2}} \\ (\text{if } |f-f_c| \leq f_D) \\ 0\ (\text{else}) \end{cases}$$

Combined Characteristics

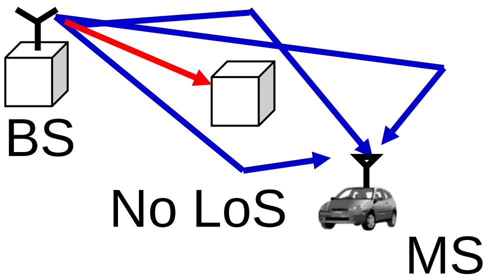
- Combined path loss, shadowing and narrowband fading





Rayleigh Distribution

- For any 2 Gaussian random variables X and Y, both with zero mean and equal variance σ^2



$$z(t) = |r(t)| = \sqrt{r_I^2(t) + r_Q^2(t)}$$

  - $Z = \sqrt{X^2 + Y^2}$: Rayleigh distributed

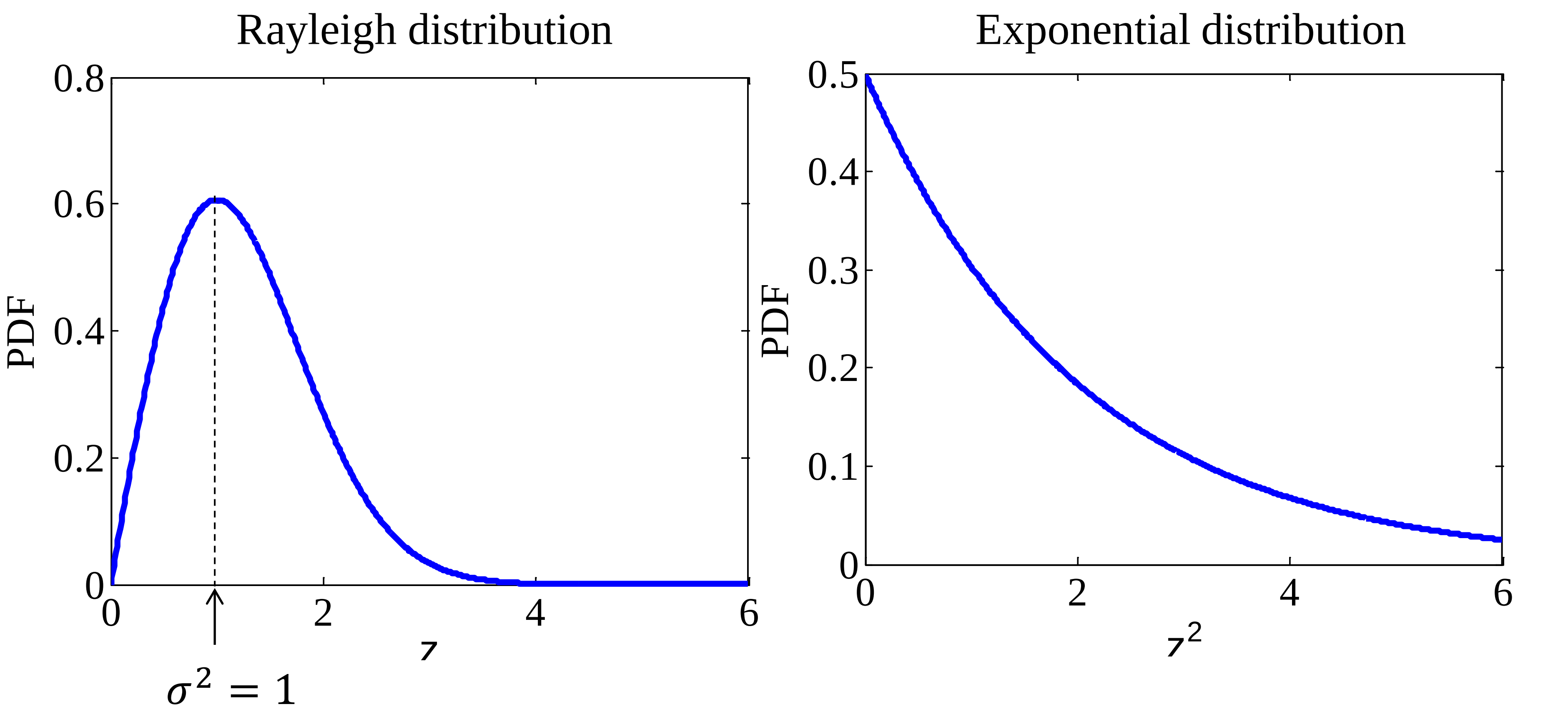
$$f_Z(z) = \frac{2z}{\bar{P}_r} \exp\left[-\frac{z^2}{\bar{P}_r}\right] = \frac{z}{\sigma^2} \exp\left[-\frac{z^2}{2\sigma^2}\right] \quad z \geq 0$$

$$\text{Mean power: } \bar{P}_r = \sum_n E[\alpha_n^2] = 2\sigma^2$$

  - Z^2 : exponentially distributed

$$f_{Z^2}(x) = \frac{1}{\bar{P}_r} \exp\left[-\frac{x}{\bar{P}_r}\right] = \frac{1}{2\sigma^2} \exp\left[-\frac{x}{2\sigma^2}\right] \quad x \geq 0$$

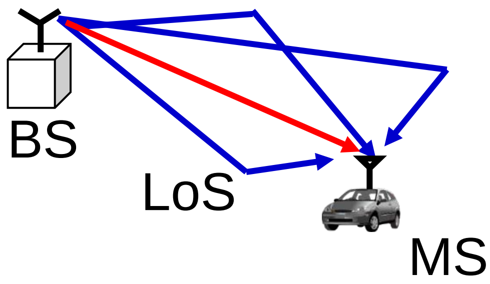
Rayleigh Distribution



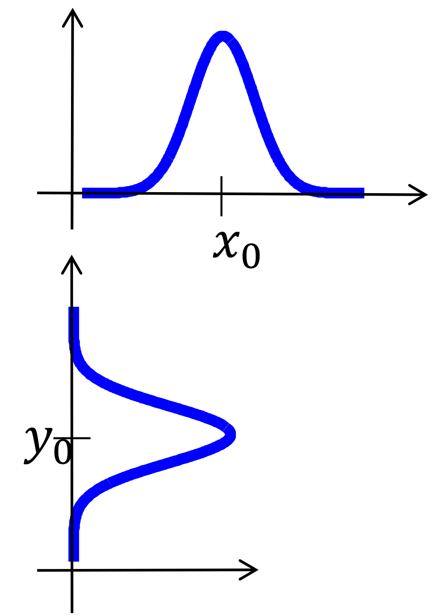
Rice Distribution

- If channel has fixed LOS component, then I and Q components are not zero-mean variables
- Superposition of complex Gaussian component and LOS component

  → Envelope is Rician distributed



$$f_Z(z) = \frac{z}{\sigma^2}\exp\left[-\frac{(z^2+s^2)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right) \qquad z \geq 0$$



$\sum_{n,n\neq 0} E[\alpha_n^2] = 2\sigma^2$: Non-LoS components

$E[\alpha_0^2] = s^2 = x_0^2 + y_0^2$: LoS components

I_0: modified Bessel function of zeroth order

Rice Distribution

- K factor

$$K = \frac{s^2}{2\sigma^2}$$

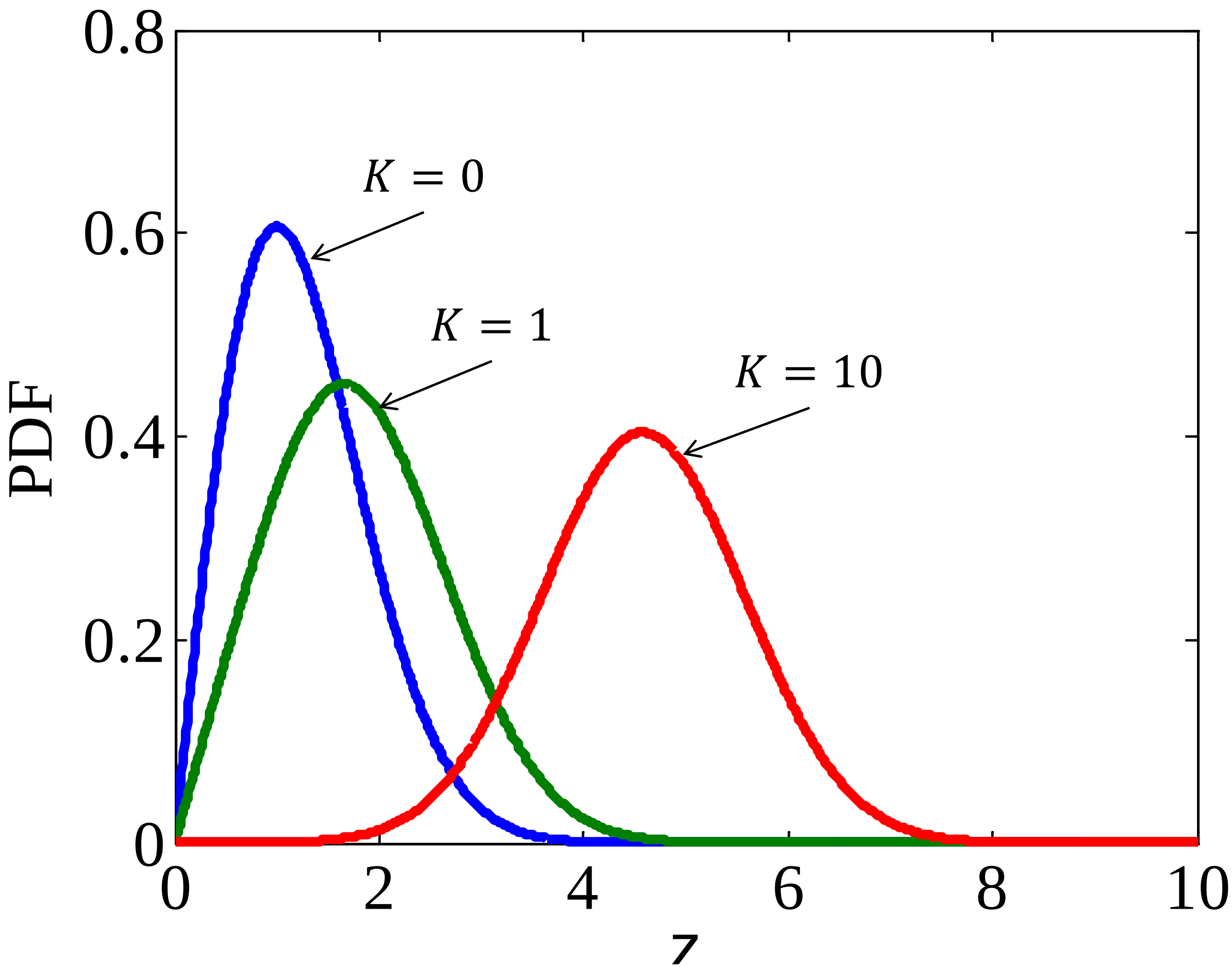
$K = \infty$ when $\sigma = 0$ (pure LoS; No fading)
$K = 0$ when $s = 0$ (pure NLoS; Rayleigh fading)

- Rice distribution expressed by K factor

$$f_Z(z) = \frac{2z(K+1)}{\overline{P}_r} \exp\left[-K - \frac{(K+1)z^2}{\overline{P}_r}\right] I_0\left(2z\sqrt{\frac{K(K+1)}{\overline{P}_r}}\right) \quad z \geq 0$$

$$\left(\begin{array}{l} \overline{P}_r = \int_0^\infty z^2 f_Z(z)dz = s^2 + 2\sigma^2 \\ s^2 = \frac{K\overline{P}_r}{K+1} \qquad 2\sigma^2 = \frac{\overline{P}_r}{K+1} \end{array}\right)$$

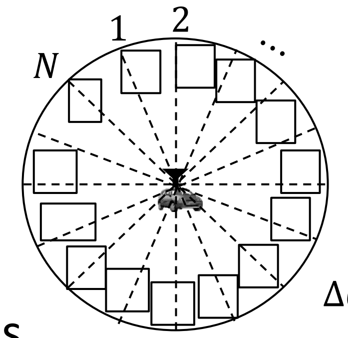
Rice Distribution



Problem

- Develop a Rayleigh fading simulator for a mobile communications channel using the following two methods (choose one)
  - Method I
    Using Clarke's model
    (Jake's Approach)

    Reference:
    Microwave Mobile Communications
    (W. C. Jakes, pp.67-69, Willey)



  - Method II
    Filtering Gaussian processes based on the in-phase and quadrature PSDs as

$$S_{r_I}(f) = S_{r_Q}(f) = \mathcal{F}\left[A_{r_I}(\tau)\right] = \begin{cases} \dfrac{2P_r}{\pi f_D} \dfrac{1}{\sqrt{1-(f/f_D)^2}} & |f| \le f_D \\ 0 & \text{else} \end{cases}$$

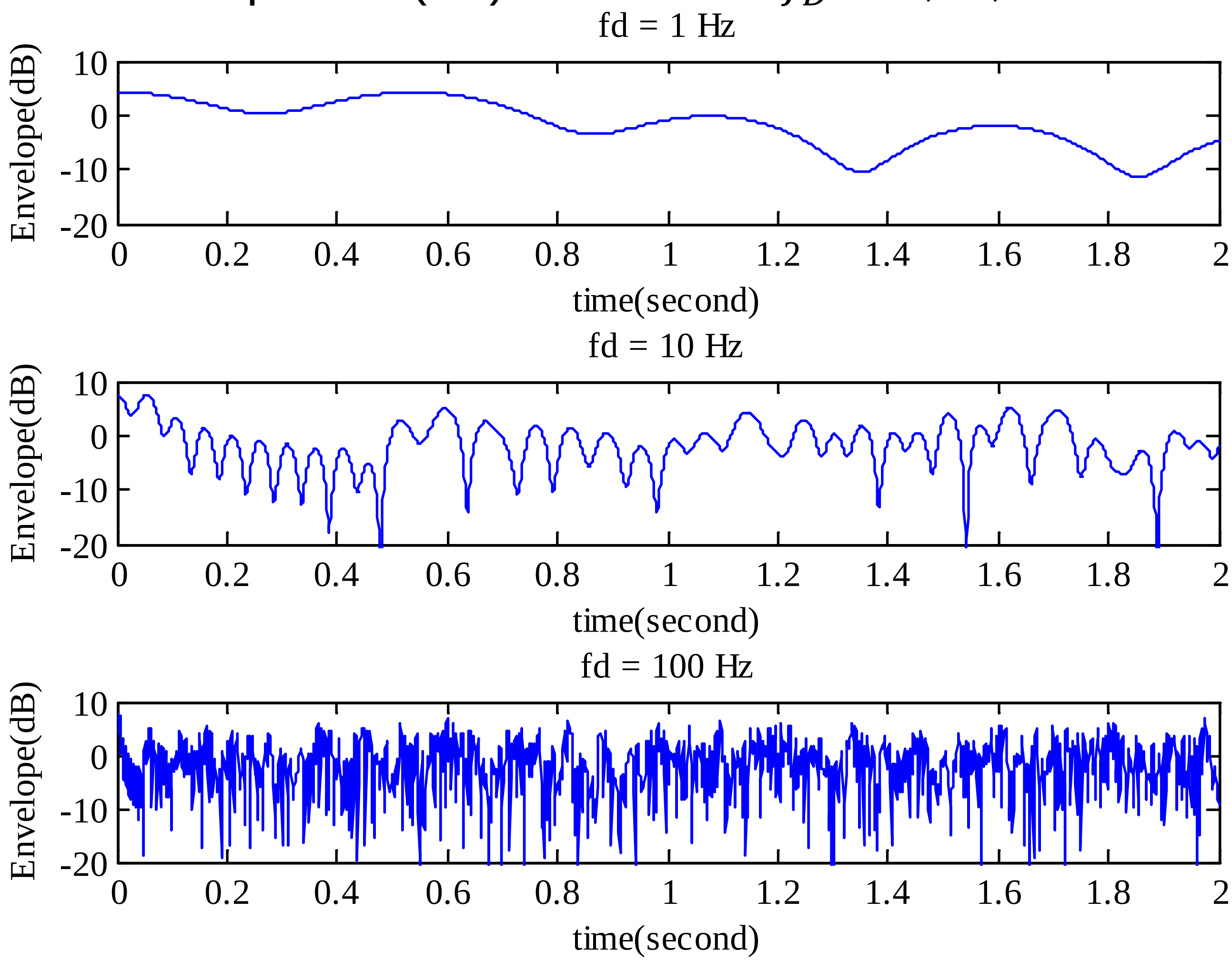
Problem

- In this problem, you must do the following
  - Develop simulation code to generate a signal with Rayleigh fading amplitude over time. Your sample rate should be at least 1000 samples/sec, the average received envelope should be 1, and your simulation should be parameterized by the Doppler frequency f_D.
  - Write a description of your simulation that clearly explains how your code generates the fading envelope using a block diagram and any necessary equations.
  - Provide plots of received amplitude (dB) vs. time for $f_D = 1,10,100$ Hz (see the next page)
  - Provide plots of empirical PDFs of the envelope and power, autocorrelation function, and PSD function

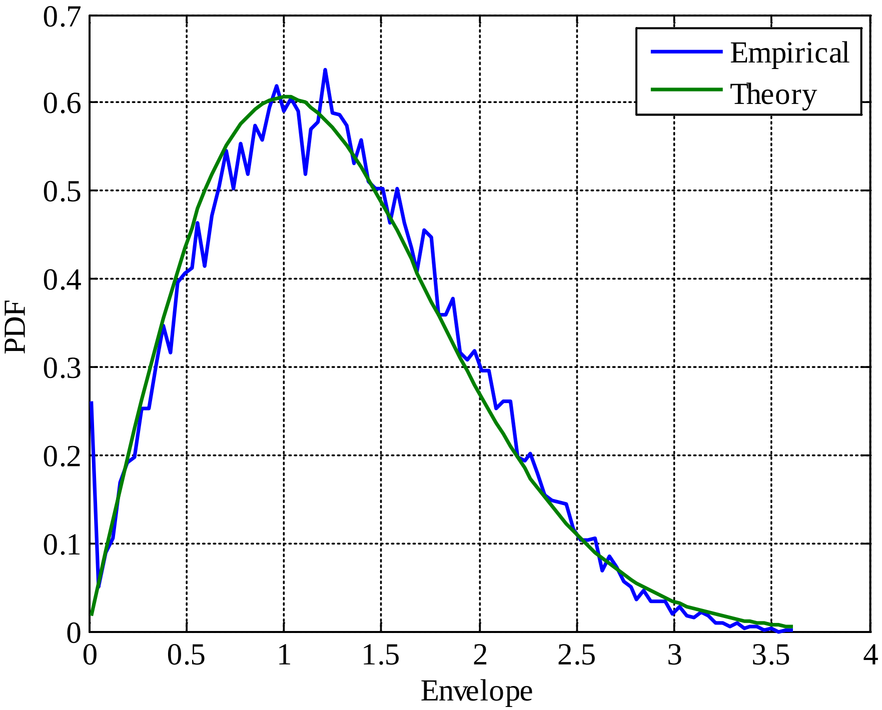
提出日：7月19日（月）

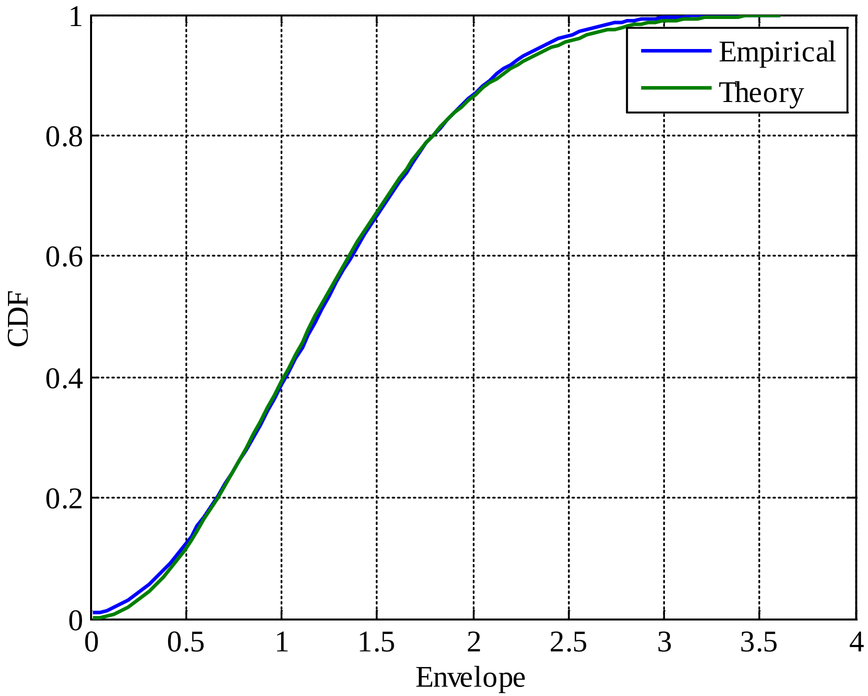
Example: Envelope

- Received amplitude (dB) vs. time for f_D = 1,10,100 Hz

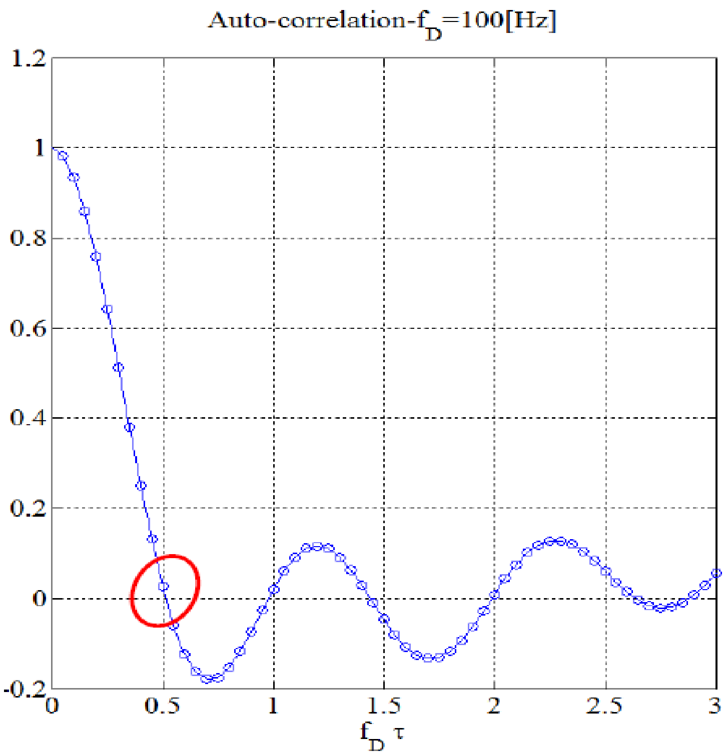


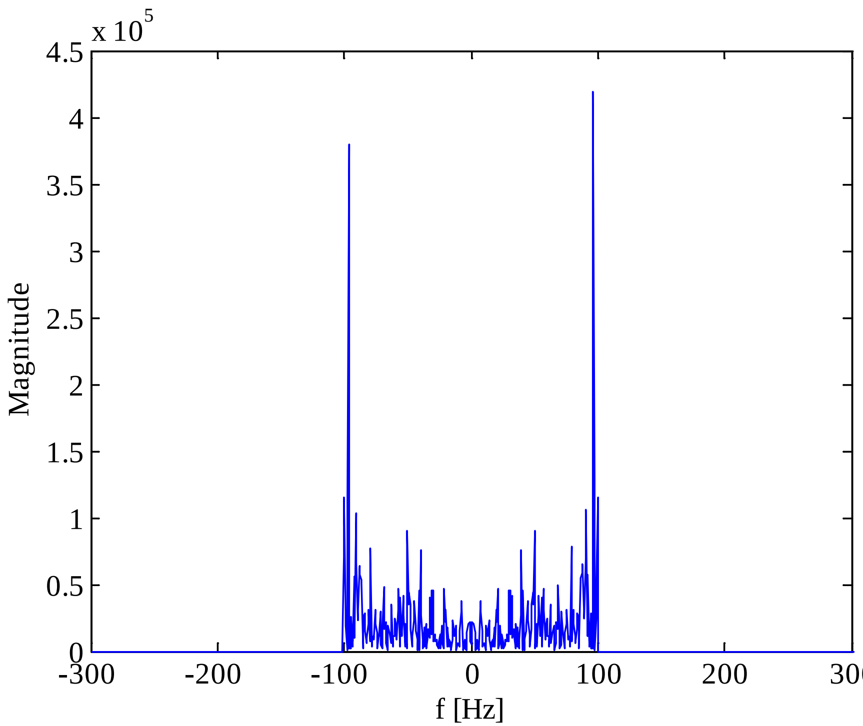
Example : Empirical PDF & CDF





Example : Autocorrelation & PSD



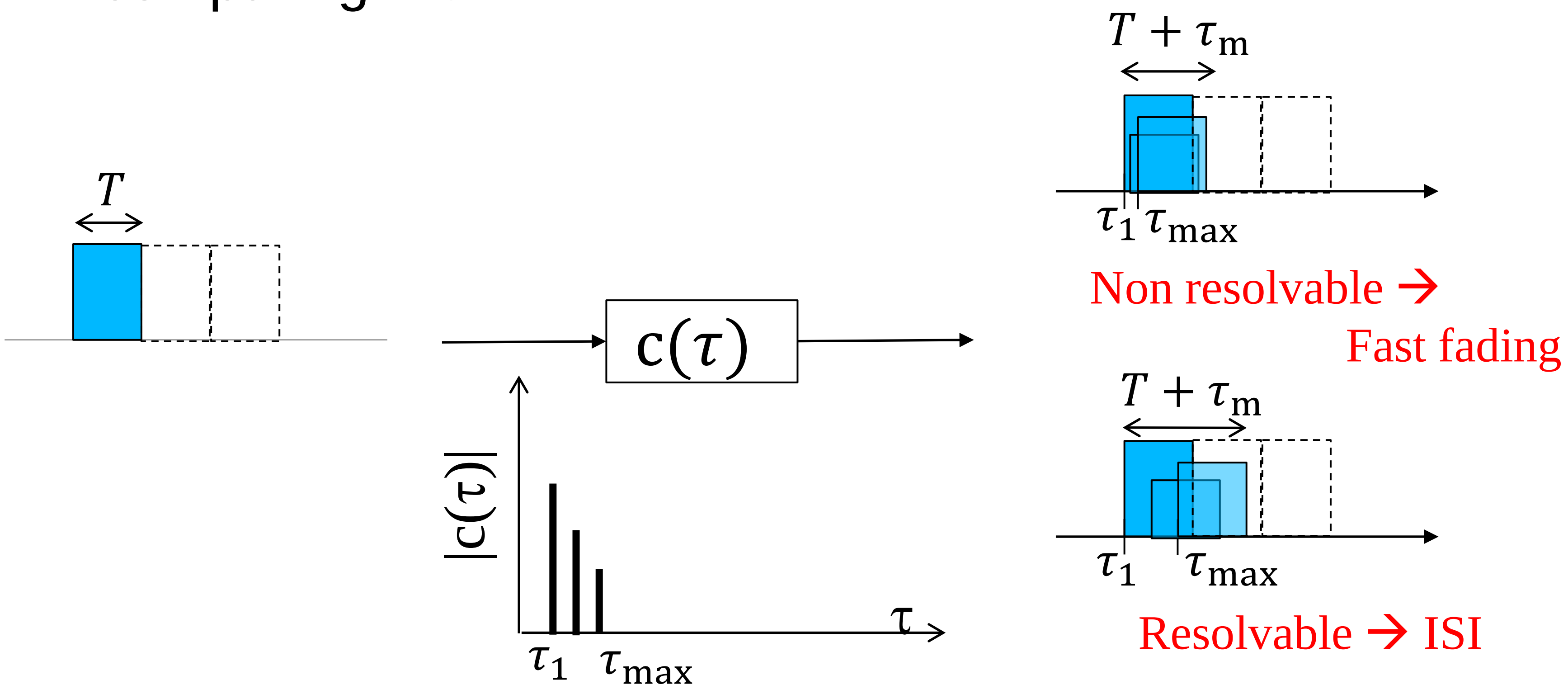


Outline

- Statistical Multipath Channel Model
  - Mathematical Representation
  - Narrowband Fading Model
  - Wideband Fading Model

Narrowband or Wideband Fading

- If the signal is not narrowband, it is also distorted by multipath delay spread → Wideband fading
- ISI (inter-symbol-interference) occur when τ_m is large comparing with T



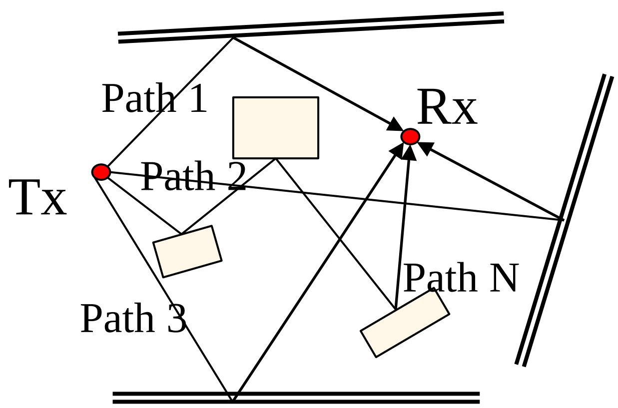
Wideband Fading

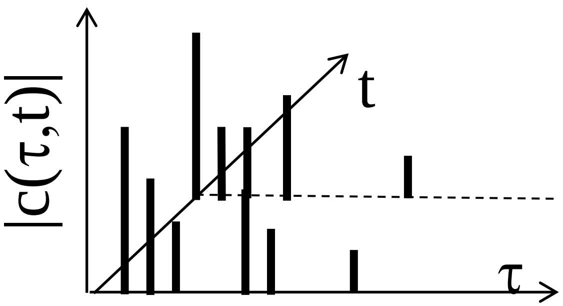
- When the signal bandwidth increases ($\tau_{\mathrm{m}} \approx B^{-1}$), the approximation $u(t-\tau_i) \approx u(t)$ is no longer valid
- Fading mitigation
  - Narrowband fading
    - Diversity
  - Wideband fading
    - Equalization
    - Multicarrier modulation
    - Spread spectrum

Wideband Fading

- The effect of multipath on wideband signals must be considered by *delay spread* and *time variation* → Statistical characterization of $c(\tau,t)$
- Time varying channel impulse response

$$c(\tau,t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\varphi_n(t)} \delta\big(\tau - \tau_n(t)\big)$$

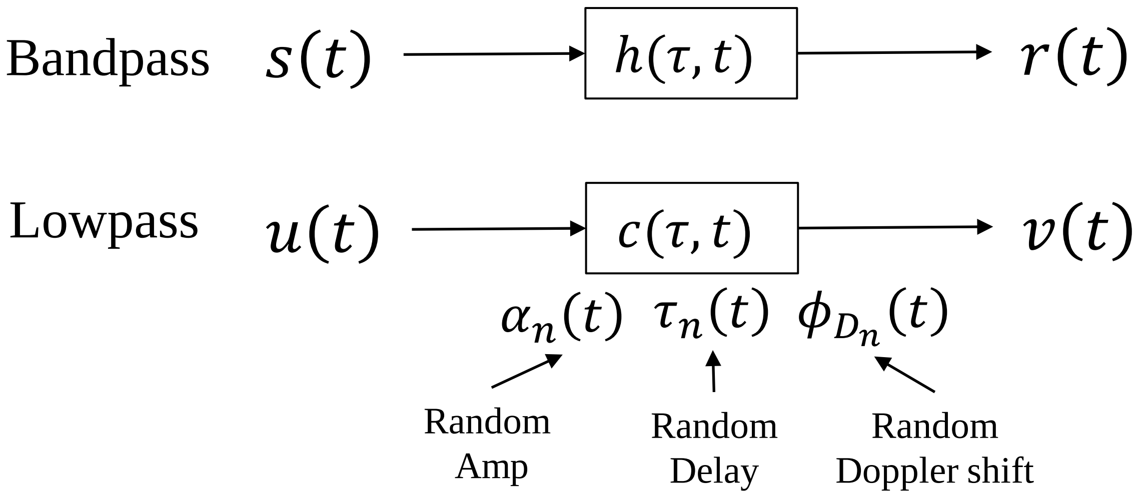




Channel impulse response

Remind: Signal Model

- Signal representation



$$s(t) = A\cos\big(2\pi f_c t + \theta(t)\big) = \mathrm{Re}\big[u(t)e^{\mathrm{j}2\pi f_c t}\big]$$

$$r(t) = \mathrm{Re}\left[\sum_{n=1}^{N(t)} \alpha_n(t) u\big(t - \tau_n(t)\big) e^{\mathrm{j}\left(2\pi f_c\left(t-\tau_n(t)\right)+\phi_{D_n}(t)\right)}\right]$$

Remind: Signal Model

- Received signal

$$r(t) = \mathrm{Re}\left[\sum_{n=1}^{N(t)} \alpha_n(t) u\big(t-\tau_n(t)\big) e^{\mathrm{j}\left(2\pi f_c (t-\tau_n(t)) + \phi_{D_n}(t)\right)}\right]$$

$$= \mathrm{Re}\left[\int_{-\infty}^{\infty} c(\tau,t) u(t-\tau) d\tau \cdot e^{\mathrm{j}2\pi f_c t}\right]$$

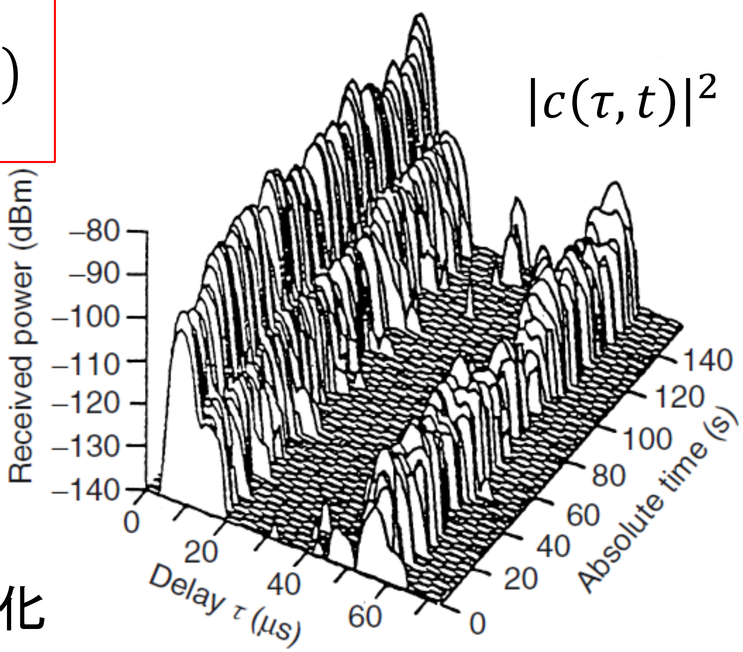
$$\boxed{c(\tau,t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-\mathrm{j}\varphi_n(t)} \delta\big(\tau-\tau_n(t)\big)}$$

- Equivalent baseband representation

$$v(t) = \int_{-\infty}^{\infty} \underline{c(\tau,t)} u(t-\tau) d\tau$$

遅延広がりが時間的に変化

$|c(\tau,t)|^2$



Autocorrelation function

- Autocorrelation function assuming WSSUS

$$A_c(\tau_1, \tau_2; t, t+\Delta t) = E[c^*(\tau_1; t)c(\tau_2; t+\Delta t)]$$

$$\Big\downarrow \text{ Wide sense stationary (WSS)}$$

$$A_c(\tau_1, \tau_2; \Delta t) = E[c^*(\tau_1; t)c(\tau_2; t+\Delta t)]$$

$$\Big\downarrow \text{ Uncorrelated scattering (US)}$$

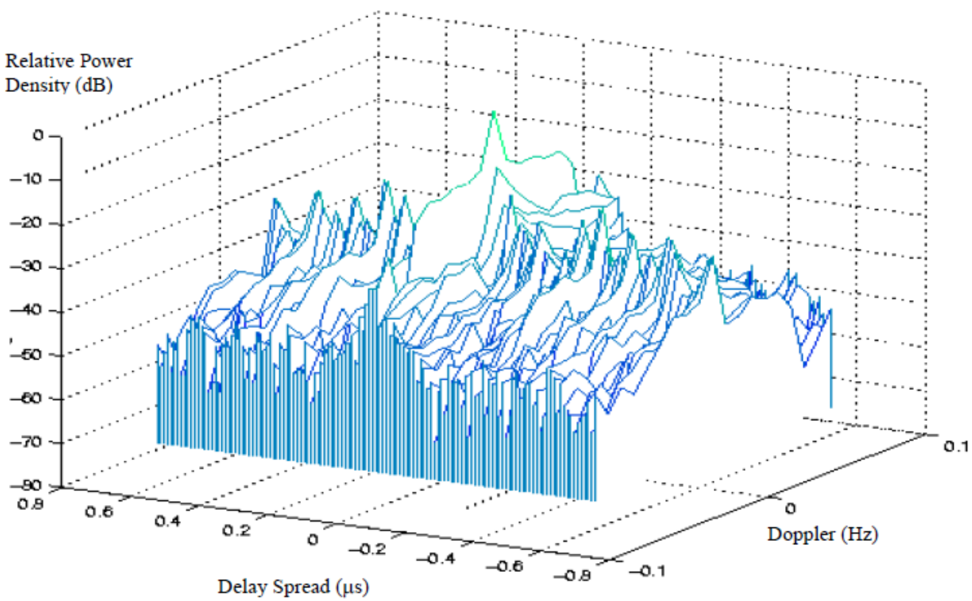
$$A_c(\tau_1, \tau_2; \Delta t) = A_c(\tau_1; \Delta t)\delta(\tau_1 - \tau_2) \triangleq \boxed{A_c(\tau; \Delta t)}$$

  - WSS: joint statistics of a channel measured at two different times t and $t + \Delta t$ depends only on the time difference Δt
  - US: the channel response associated with a given multipath component of delay τ_1 is uncorrelated with that of delay $\tau_2 \neq \tau_1$

Scattering function

- Scattering function characterizes the average output power associated with the channel as a function of the multipath delay τ and Doppler ρ

$$S_c(\tau,\rho) = \int_{-\infty}^{\infty} A_c(\tau;\Delta t)\, e^{-j2\pi\rho\Delta t} d\Delta t$$



Power Delay Profile

- Power delay profile (or multipath intensity profile)

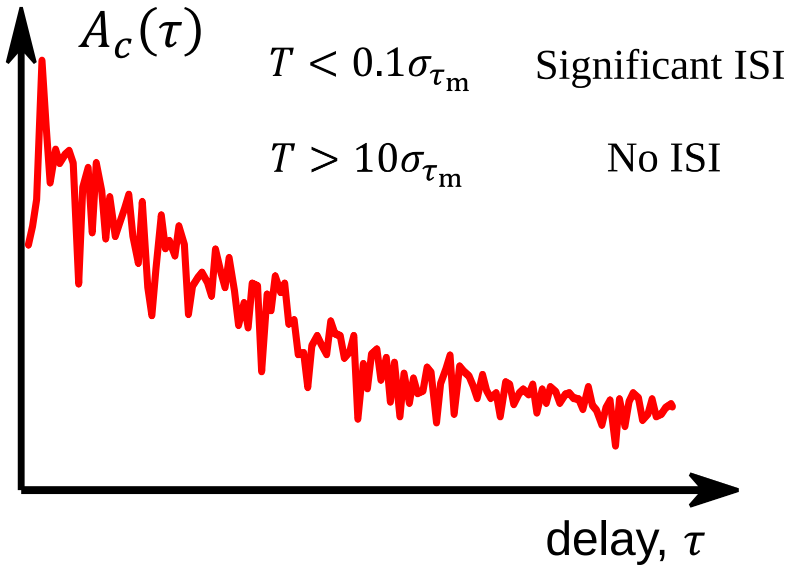
$$A_c(\tau;\Delta t=0)=\boxed{E[|c(\tau;t)|^2]=A_c(\tau)}$$

Real number function
indicating average power associated with a given multipath delay

- RMS delay spread

$$\sigma_{\tau_m}=\sqrt{\frac{\int_{-\infty}^{\infty}(\tau-\mu_{\tau_m})^2A_c(\tau)d\tau}{\int_{-\infty}^{\infty}A_c(\tau)d\tau}}$$

where $\mu_{\tau_m}=\frac{\int_{-\infty}^{\infty}\tau A_c(\tau)d\tau}{\int_{-\infty}^{\infty}A_c(\tau)d\tau}$



Frequency Domain Characterization

- Time varying transfer function

$$C(f;t)=\int_{-\infty}^{\infty} c(\tau;t)\, e^{-\mathrm{j}2\pi f\tau} d\tau$$

- Autocorrelation function

$$A_C(f_1,f_2;\Delta t)=E[C^*(f_1;t)C(f_2;t+\Delta t)]$$

$$=E\left[\int_{-\infty}^{\infty} c^*(\tau_1;t)\, e^{\mathrm{j}2\pi f_1\tau_1} d\tau_1 \int_{-\infty}^{\infty} c(\tau_2;t+\Delta t)\, e^{-\mathrm{j}2\pi f_2\tau_2} d\tau_2\right]$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[c^*(\tau_1;t)c(\tau_2;t+\Delta t)]e^{\mathrm{j}2\pi f_1\tau_1}e^{-\mathrm{j}2\pi f_2\tau_2}d\tau_1 d\tau_2$$

WSSUS $\Rightarrow$

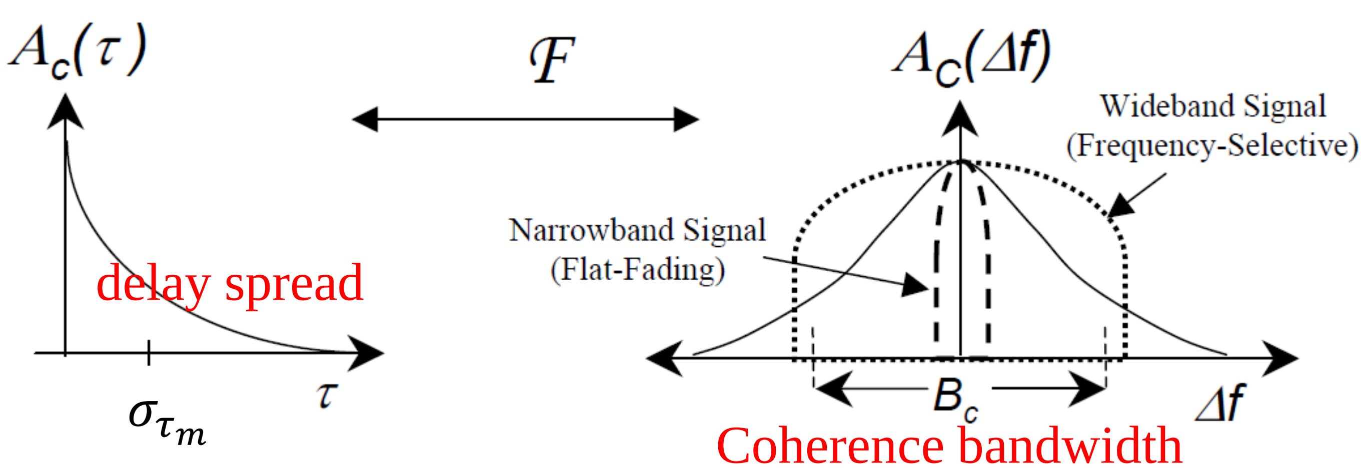
$$=\int_{-\infty}^{\infty} A_c(\tau;\Delta t)e^{-\mathrm{j}2\pi(f_2-f_1)\tau}d\tau = A_C(\Delta f;\Delta t)$$

Coherence Bandwidth

- Frequency autocorrelation function

$$A_C(\Delta f) \triangleq A_C(\Delta f; \Delta t = 0) = \int_{-\infty}^{\infty} A_c(\tau)\, e^{-j2\pi\Delta f\tau} d\tau$$

- Coherence bandwidth (B_c): $B_c \approx k \dfrac{1}{\sigma_{\tau_m}}$
  - Flat fading for $B \ll B_c$ ($T > 10\sigma_{\tau_m}$)
  - Frequency selective fading for $B \gg B_c$ ($T < 0.1\sigma_{\tau_m}$)

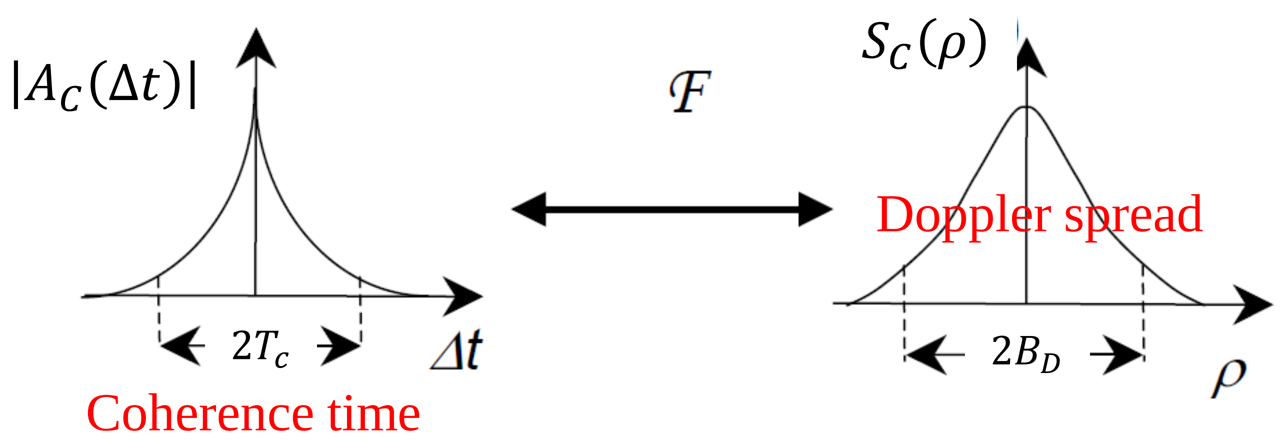


Doppler Power Spectrum & Coherence Time

- Doppler power spectrum characterizing time variation

$$S_C(\rho) \triangleq S_C(\Delta f = 0, \rho) = \int_{-\infty}^{\infty} A_C(\Delta f = 0; \Delta t)\, e^{-j2\pi\rho\Delta t} d\Delta t$$

- Coherence time (T_c): $T_c \approx k \frac{1}{B_D}$
  - Slow fading for $T \ll T_c$ ($B \gg B_D$) → シンボル周期に比べてコヒーレンス時間が十分長い
  - Fast fading for $T > T_c$ ($B < B_D$) → シンボル周期に比べてコヒーレンス時間が短い



Fourier Relationship

- Time varying channel impulse response (CIR) & transfer function (TF)

$$C(f,t) = \int_{-\infty}^{\infty} c(\tau;t)\, e^{-j2\pi f\tau} d\tau$$

- Autocorrelation

$$A_C(\Delta f;\Delta t) = \int_{-\infty}^{\infty} A_c(\tau;\Delta t) e^{-j2\pi\Delta f\tau} d\tau$$

where $A_c(\tau;\Delta t) = E[c^*(\tau_1;t)c(\tau_2;t+\Delta t)]$ and

$$A_C(\Delta f;\Delta t) = E[C^*(f;t)C(f+\Delta f;t+\Delta t)]$$

- Scattering function

$$S_c(\tau,\rho) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_C(\Delta f;\Delta t)\, e^{-j2\pi\rho\Delta t} e^{j2\pi\tau\Delta f} d\Delta t d\Delta f$$

Summary

$A_c(\tau) = A_c(\tau; \Delta t = 0)$ Power delay profile

$$A_c(\tau; \Delta t)$$

Autocorr. of CIR

Freq. correlation function

$A_C(\Delta f) =$
$A_C(\Delta f; \Delta t = 0)$

$\mathcal{F}$ $\mathcal{F}^{-1}$

$\mathcal{F}^{-1}$ $\mathcal{F}$

$$A_C(\Delta f; \Delta t)$$

Autocorr. of TF

$$S_c(\tau, \rho)$$

Scattering function (delay-Doppler spread)

$A_C(\Delta t) =$
$A_C(\Delta f = 0; \Delta t)$

Time correlation function

$\mathcal{F}$ $\mathcal{F}^{-1}$

$\mathcal{F}$ $\mathcal{F}^{-1}$

$$S_C(\Delta f; \rho)$$

$S_C(\rho) = S_C(\Delta f = 0; \rho)$ Doppler power spectrum

Summary

- System bandwidth
  - Narrowband fading
  - Wideband fading
- Frequency correlation
  - Flat fading
  - Frequency selective fading
- Time correlation
  - Slow fading
  - Fast fading