電子情報通信特論
Advanced Course of Electronics, Information and Communications

Path Loss and Shadowing

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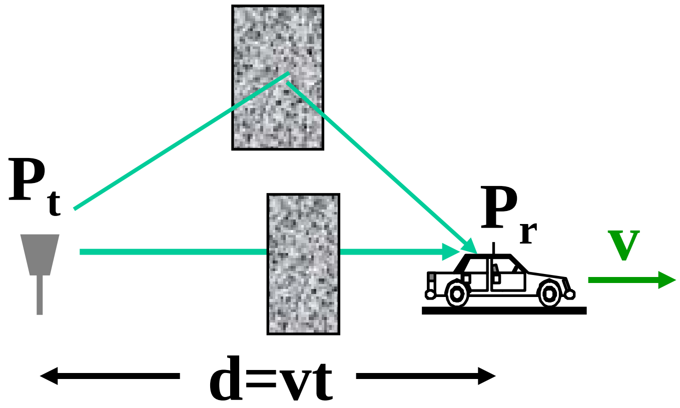
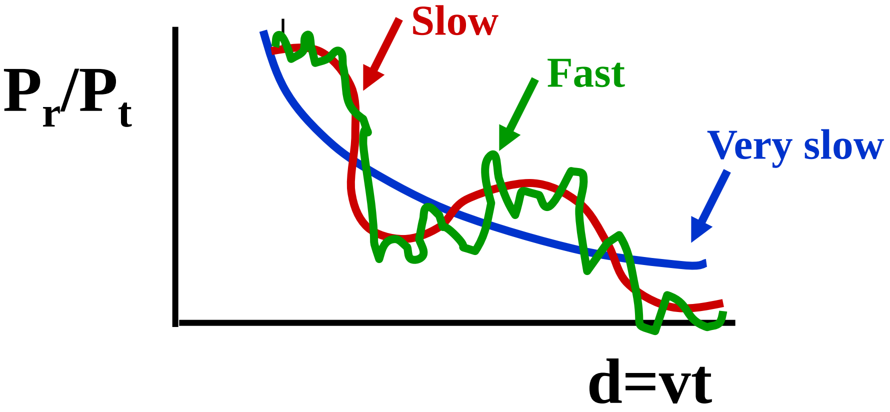
Radio Propagation

- The variation can be categorized into two kinds:
  - Large scale propagation effects :
    - Path Loss (includes average shadowing)
    - Shadowing (due to obstructions)
  - Small scale propagation effects: constructive or destructive addition of multipath signal components → multipath fading

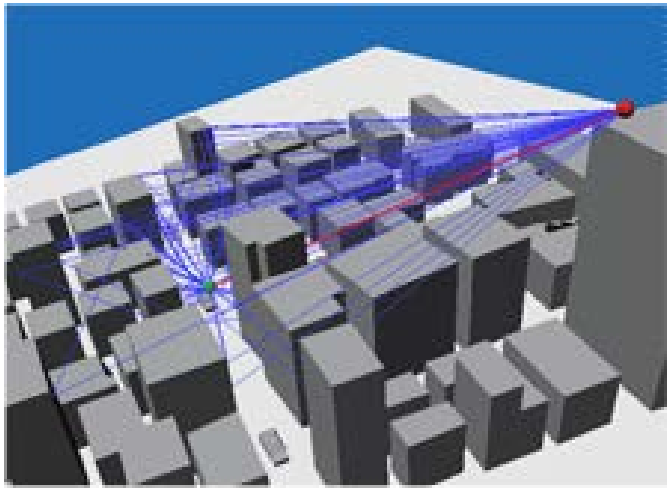




Radio Propagation

- Electromagnetic wave propagation can be characterized by solving the Maxwell's equation
  - Including reflection, scattering, and diffraction
  - Boundary condition
  - Complicated calculations
- Approximation was employed to avoid this difficulty, such as by introducing Ray Tracing technique

幾何光学



Transmit and Receive Signal Models

- Tx and Rx signals considered are "real"
  - Modulators are built using oscillators that generate real sinusoids
  - Signals are modeled using complex representation for analytical simplicity
- Narrowband (CW) case

Transmitted signal: $s(t) = 1 \cdot \cos 2\pi f_c t \rightarrow \mathrm{Re}\left[1 \cdot e^{\mathrm{j}2\pi f_c t}\right]$

Received signal: $r(t) = \alpha \cos 2\pi f_c (t - \Delta t) \rightarrow \mathrm{Re}\left[\alpha e^{-\mathrm{j}\phi} \cdot e^{\mathrm{j}2\pi f_c t}\right]$

$$\phi = 2\pi f_c \Delta t$$

Real (Bandpass) $\quad s(t) \longrightarrow \boxed{h(t)} \longrightarrow r(t)$

Complex (Lowpass) $\quad 1 \longrightarrow \boxed{\alpha e^{-\mathrm{j}\phi}} \longrightarrow \alpha e^{-\mathrm{j}\phi}$

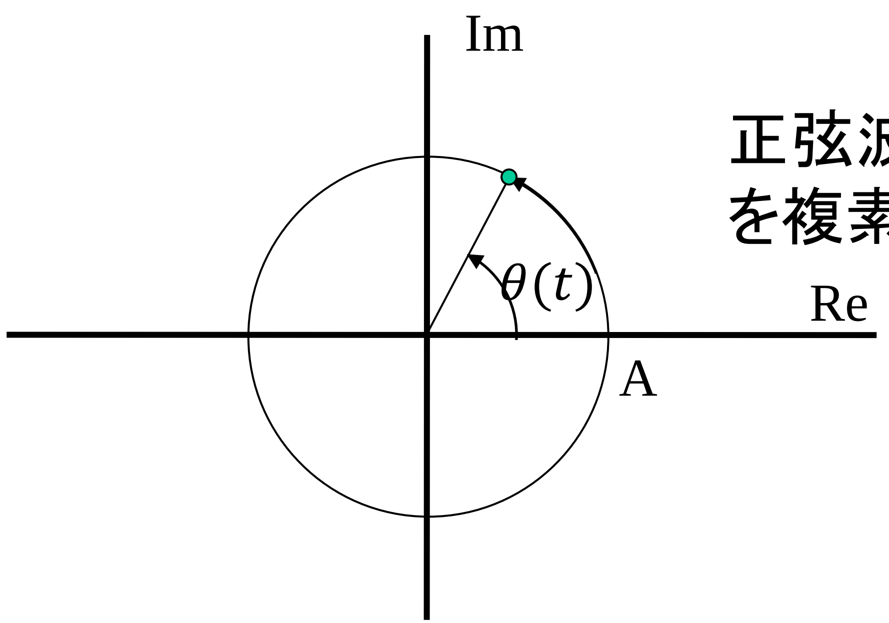
Modulated Signal

- Real modulated signals are often represented as the real part of a complex signal
- Bandpass signal (real signal)

$$s(t) = A\cos(2\pi f_c t + \theta(t)) = \mathrm{Re}\left[Ae^{\mathrm{j}\theta(t)}e^{\mathrm{j}2\pi f_c t}\right]$$

- (Equivalent) lowpass signal (complex envelope)

$$u(t) = Ae^{\mathrm{j}\theta(t)} = A(\cos\theta(t) + \mathrm{j}\sin\theta(t))$$



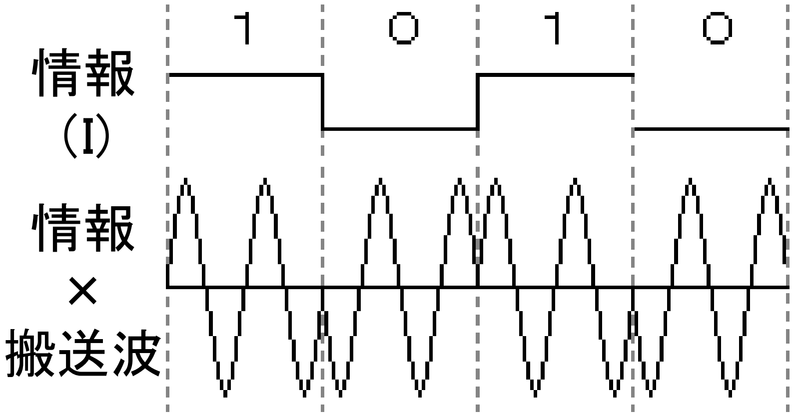
正弦波の振幅と位相（情報）を複素平面で表現

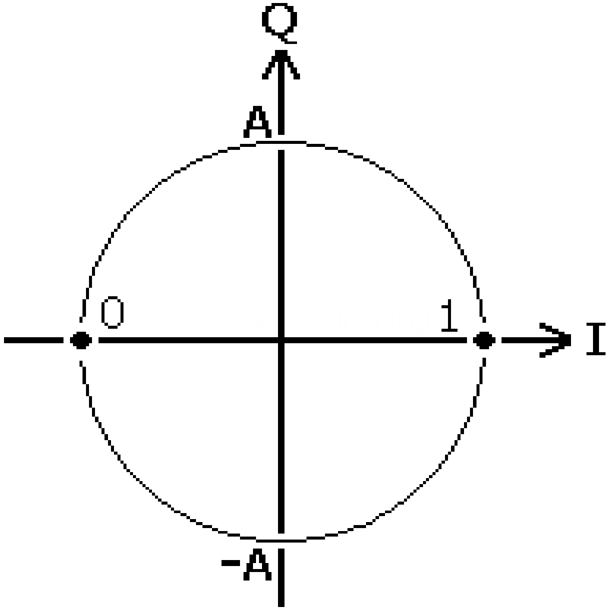
Digital Modulation

- BPSK (binary phase shift keying)

$$u(t) = Ae^{\mathrm{j}\theta(t)} = \begin{cases} A & (I=1) \\ -A & (I=0) \end{cases}$$

$$s(t) = \mathrm{Re}\left[e^{\mathrm{j}2\pi f_c t} A e^{\mathrm{j}\theta(t)}\right] = \begin{cases} A\cos 2\pi f_c t & (I=1) \\ -A\cos 2\pi f_c t & (I=0) \end{cases}$$





Bandpass Signal Generation

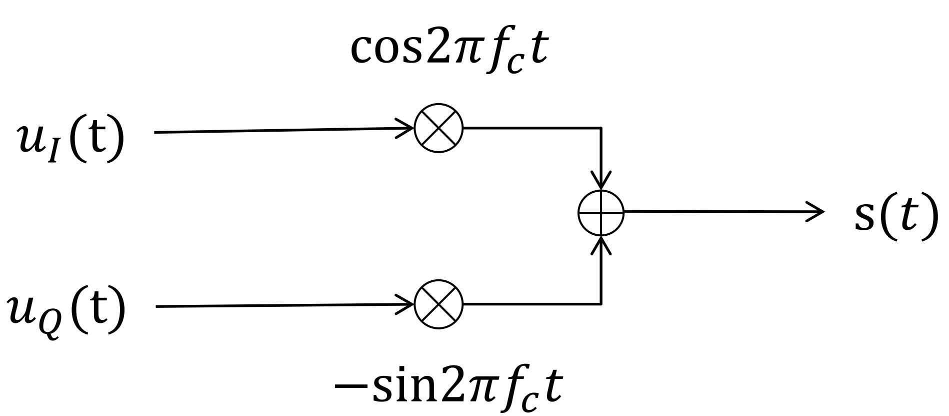
- Equivalent lowpass signal (complex envelope)

$$u(\mathrm{t}) = Ae^{\mathrm{j}\theta(t)} = A(\cos\theta(t) + \mathrm{j}\sin\theta(t)) = u_I(t) + \mathrm{j}u_Q(t)$$

$$s(t) = A\cos(2\pi f_c t + \theta(t)) = \mathrm{Re}\left[u(t)e^{\mathrm{j}2\pi f_c t}\right]$$

$$= \mathrm{Re}\left[\left(u_I(t) + \mathrm{j}u_Q(t)\right)(\cos 2\pi f_c t + \mathrm{j}\sin 2\pi f_c t)\right]$$

$$= u_I(\mathrm{t})\cos 2\pi f_c t - u_Q(t)\sin 2\pi f_c t$$ バンドパス信号



Complex Baseband Signal

- Bandpass signal

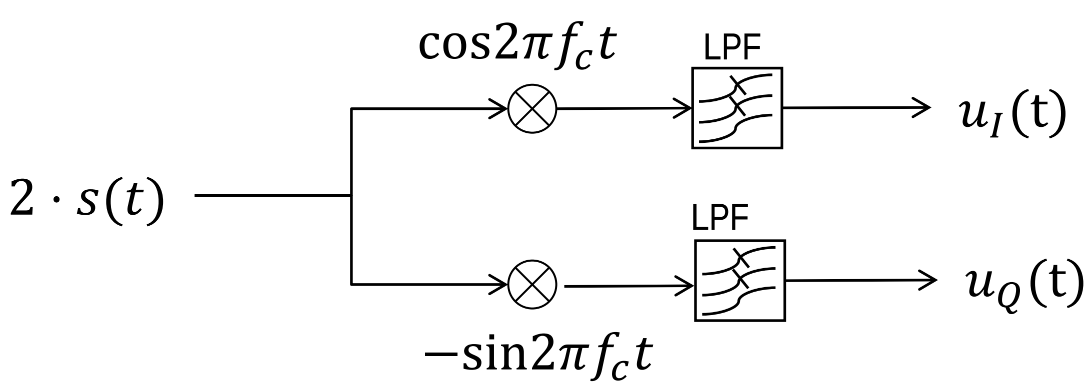
$$s(t) = \mathrm{Re}\left[u(t)e^{\mathrm{j}2\pi f_c t}\right] = \frac{1}{2}\left(u(t)e^{\mathrm{j}2\pi f_c t} + u^*(t)e^{-\mathrm{j}2\pi f_c t}\right)$$

- Frequency shift

$$e^{-\mathrm{j}2\pi f_c t} \cdot 2 \cdot s(t) = u(t) + u^*(t)e^{-\mathrm{j}4\pi f_c t}$$

- Lowpass filtering

$$\mathrm{LPF}\left[2e^{-\mathrm{j}2\pi f_c t}s(t)\right] = \mathrm{LPF}\left[u(t) + u^*(t)e^{-\mathrm{j}4\pi f_c t}\right] = u(t)$$



Transmit and Receive Signal Models (2)

- Transmitted signal

$$s(t) = A(t)\cos(2\pi f_c t + \theta(t)) = \mathrm{Re}\left[u(t)e^{\mathrm{j}2\pi f_c t}\right]$$

where baseband signal (digitally modulated) is denoted by

$$u(t) = u_I(t) + \mathrm{j}u_Q(t) = A(t)e^{\mathrm{j}\theta(t)}$$

- Channel impulse response

$$h(t) = \mathrm{Re}\left[c(t)e^{\mathrm{j}2\pi f_c t}\right]$$

- Received signal

$$r(t) = h(t) * s(t) + n(t) = \underline{\mathcal{F}^{-1}\{H(f)S(f)\}} + n(t)$$

$$\frac{1}{4}\mathcal{F}^{-1}\{[C(f-f_c) + C^*(-f-f_c)][U(f-f_c) + U^*(-f-f_c)]\}$$

$$= \frac{1}{4}\mathcal{F}^{-1}\{C(f-f_c)U(f-f_c) + C^*(-f-f_c)U^*(-f-f_c)\}$$

$$= \frac{1}{2}\mathrm{Re}\left[v(t)e^{\mathrm{j}2\pi f_c t}\right]$$

$\therefore v(t) = c(t) * u(t) \rightarrow$ equivalent lowpass signal model

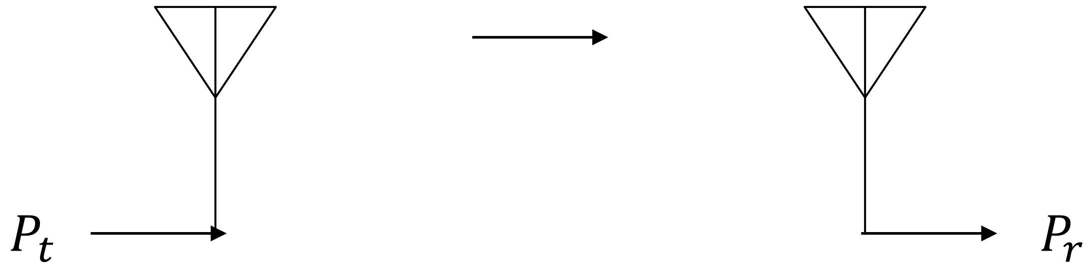
Path Loss

- Path Loss (already negative meaning)

$$L = \frac{P_t}{P_r} \rightarrow P_L[\mathrm{dB}] = 10\log_{10}\frac{P_t}{P_r} \text{ (in decibels)}$$

- Path Gain

$$P_G = \frac{P_r}{P_t} = \frac{1}{P_L} \rightarrow P_G[\mathrm{dB}] = 10\log_{10}\frac{P_r}{P_t} = -L[\mathrm{dB}]$$



Free Space Path Loss

- Received signal

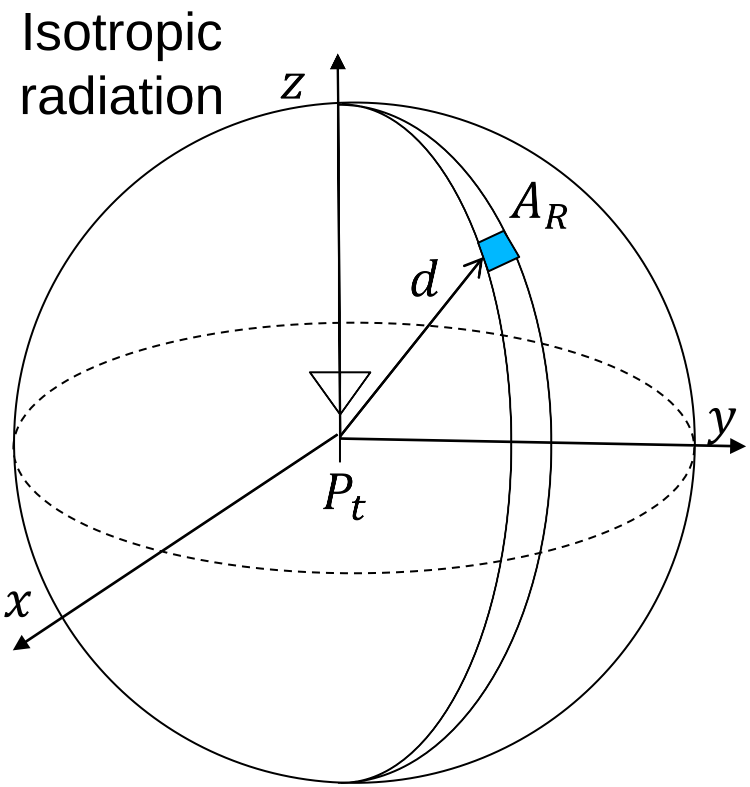
$$r(t) = \mathrm{Re}\left\{c(t) * u(t)e^{\mathrm{j}2\pi f_c t}\right\}$$
$$= \mathrm{Re}\left\{\frac{\lambda\sqrt{G_l}e^{-\mathrm{j}2\pi\frac{d}{\lambda}}}{4\pi d}u(t)e^{\mathrm{j}2\pi f_c t}\right\}$$

Antenna power gain
$G_l = G_{a,tx}G_{a,rx}$

- Free Space Path Loss

$$L = \frac{P_t}{P_r} = \left(\frac{\lambda\sqrt{G_l}}{4\pi d}\right)^{-2} = \left(\frac{\lambda^2}{(4\pi d)^2}G_{a,tx}G_{a,rx}\right)^{-1}$$

Free Space Path Loss



- The power density with distance d away from the transmitter

$$S(d) = \frac{P_t}{4\pi d^2}$$

- The incident power at the receiver antenna is determined by the antennas effective area A_R

$$P_r(d) = S(d)A_R = \frac{P_t}{4\pi d^2}A_R$$

Free Space Path Loss

- Generally, the gains of the transmit antennas are considered as

$$P_r(d) = \frac{P_t}{4\pi d^2} G_{a,tx} A_R$$

- Relationship between effective area and gain

$$A_R = \frac{\lambda^2}{4\pi} G_{a,rx}$$

- Received power

$$P_r(d) = \frac{\lambda^2}{(4\pi d)^2} G_{a,tx} G_{a,rx} P_t$$

- Path loss

$$L = \left(\frac{\lambda^2}{(4\pi d)^2} G_{a,tx} G_{a,rx} \right)^{-1}$$

Exercise

- High frequency use

  A new mobile system at higher frequency bands e.g. > 10 GHz is gaining interest for 5G networks. The received power is expressed as

  $$P_r(d) = \frac{\lambda^2}{(4\pi d)^2} G_{a,tx} G_{a,rx} P_t$$

  Now, let's consider the difference in the received powers at 6 GHz and 60 GHz.

  - If the isotropic antennas are used at Tx and Rx ($G_{a,tx} = G_{a,rx} = 1$), then how much degraded the received power at 60 GHz is ?
  - If the isotropic antenna is used only at Tx ($G_{a,tx} = 1$), how to maintain the same received power at both frequencies ?

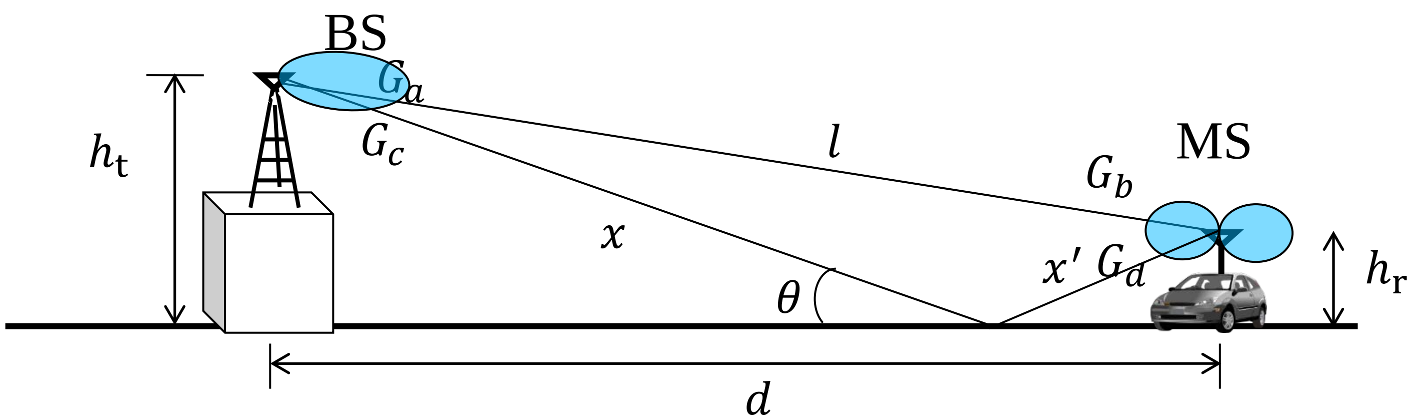
<u>Report due: May 10</u>

Ray Tracing

- Multipath effects can be found by using Maxwell's equations
  - Requires knowledge of appropriate boundary conditions
  - Impractical due to computational complexity
- Ray tracing
  - Approximate the propagation of radio waves by representing wavefronts as simple particles
  - Effects of obstacles are approximated by simple geometric equations
- Ray tracing is most accurate when:
  - Receiver is many wavelengths from nearest scatterer
  - Scatterers are large relative to wavelength
  - Scatterers are fairly smooth

Two-Ray Model

- Applicable when single ground reflection is the dominant multipath effect
- Received signal is composed of:
  - Direct LOS component
  - Ground reflected component



Two-Ray Model

- Received signal

$$r_{2-ray} = \mathrm{Re}\left\{\frac{\lambda}{4\pi}\left[\frac{\sqrt{G_l}u(t)e^{-\frac{\mathrm{j}2\pi l}{\lambda}}}{l} + \frac{R\sqrt{G_r}u(t-\tau)e^{-\frac{\mathrm{j}2\pi(x+x')}{\lambda}}}{x+x'}\right]e^{\mathrm{j}2\pi f_c t}\right\}$$

where $G_l = G_a G_b$, $G_r = G_c G_d$, and R denotes reflection coefficient

- Received power

$$P_r = P_t\left[\frac{\lambda}{4\pi}\right]^2\left|\frac{\sqrt{G_l}}{l} + \frac{R\sqrt{G_r}e^{-\mathrm{j}\Delta\phi}}{x+x'}\right|^2$$

where $\Delta\phi = 2\pi(x+x'-l)/\lambda$

Two-Ray Model

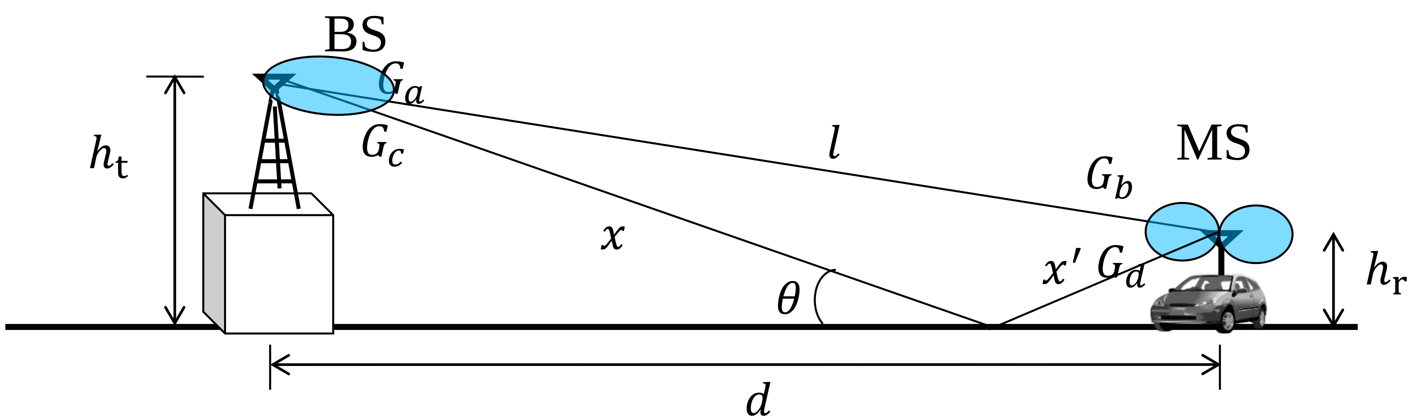
- Phase shift

$$\Delta\phi = \frac{2\pi(x + x' - l)}{\lambda} = \frac{2\pi}{\lambda}\left(\sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}\right)$$

$$\approx \frac{4\pi h_t h_r}{\lambda d} \qquad \because d \gg (h_t + h_r)$$

- Reflection coefficient

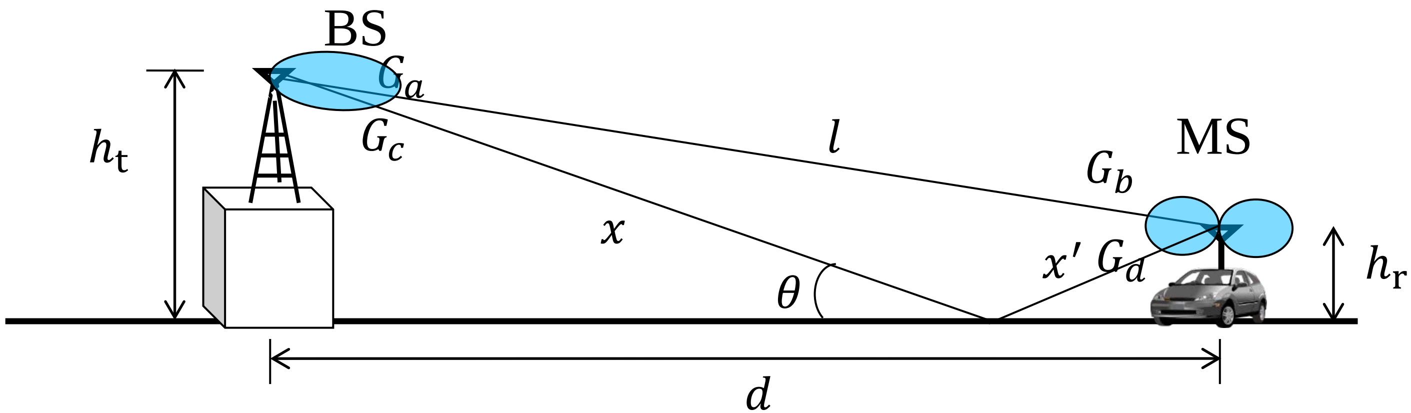
$$R = \frac{\sin\theta - Z}{\sin\theta + Z} \qquad Z = \begin{cases} \sqrt{\varepsilon_r - \cos^2\theta}\,/\varepsilon_r & \text{for V-pol} \\ \sqrt{\varepsilon_r - \cos^2\theta} & \text{for H-pol} \end{cases}$$



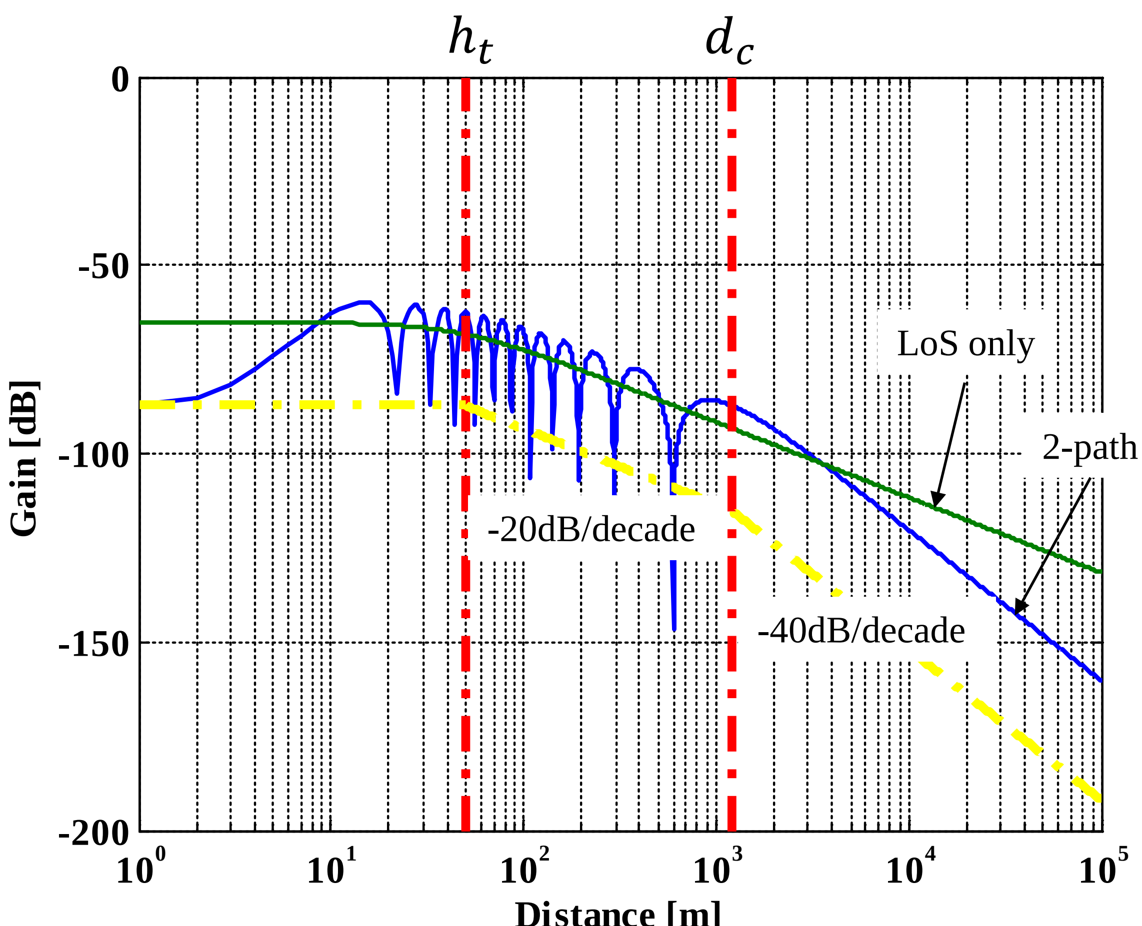
Two-Ray Model

- Approximation with large d
  $x + x' \approx l \approx d$, $\theta \approx 0$, $G_l \approx G_r$, and $R \approx -1$

$$P_r = P_t \left[\frac{\lambda}{4\pi}\right]^2 \left|\frac{\sqrt{G_l}}{l} + \frac{R\sqrt{G_r}e^{-j\Delta\phi}}{x+x'}\right|^2 \approx P_t \left[\frac{\sqrt{G_l}h_t h_r}{d^2}\right]^2$$



Two-Ray Model



$f_c = 900$ MHz
$h_t = 50$ m
$h_r = 2$ m, $G_l = G_r = 1$

- For $d < h_t$
  - Rays add constructively
- For $h_t < d < d_c$
  - Rays add constructively and destructively
  - Results in wave pattern with maxima and minima
  - Power falls off at -20 dB / decade
- For $d > d_c$
  - Rays add destructively
  - Approximation for d_c can be obtained by setting $\Delta\phi = \pi$ which results in $d_c = \frac{4h_th_r}{\lambda}$

Exercise

- Problem 2-7

  Consider a two-ray channel with impulse response $h = \alpha_1\delta(t) + \alpha_2\delta(t - 0.022\mu s)$. Find the distance separating the transmitter and receiver, as well as α_1 and α_2, assuming free-space path loss on each path with a reflection coefficient of -1. Assume the transmitter and receiver are located $8m$ above the ground and the carrier frequency is 900 MHz

Report due: May 10

Outline

- Preliminaries
  - Random Variable
  - Normal Distribution
  - Q function and Error function
- Shadow Fading
- Log-normal Distribution
- Outage Probability
- Cell Coverage Area

Random Variable (RV)

- RV is characterized by cumulative distribution function or probability density function
- Cumulative distribution function (CDF)

$$F_X(x) = \Pr\{X \leq x\} \, (-\infty < x < \infty)$$

- Probability density function (PDF)

$$f_X(x) = \frac{d}{dx} F_X(x) \qquad F_X(x) = \int_{-\infty}^{x} f_X(t)dt$$

- Properties

$$f_X(x) \geq 0$$

$$\int_{x_1}^{x_2} f_X(t)dt = F_X(x_2) - F_X(x_1) = \Pr\{x_1 \leq X \leq x_2\}$$

$$\int_{-\infty}^{\infty} f_X(t)dt = 1$$

Random Variable (RV)

- Moment

$$m_k = \int_{-\infty}^{\infty} (x-c)^k f_X(x)dx$$

- Mean and Variance
  - Mean: 1-order moment ($c = 0$)

$$E[X] = \mu = m_1 = \int_{-\infty}^{\infty} x f_X(x)dx$$

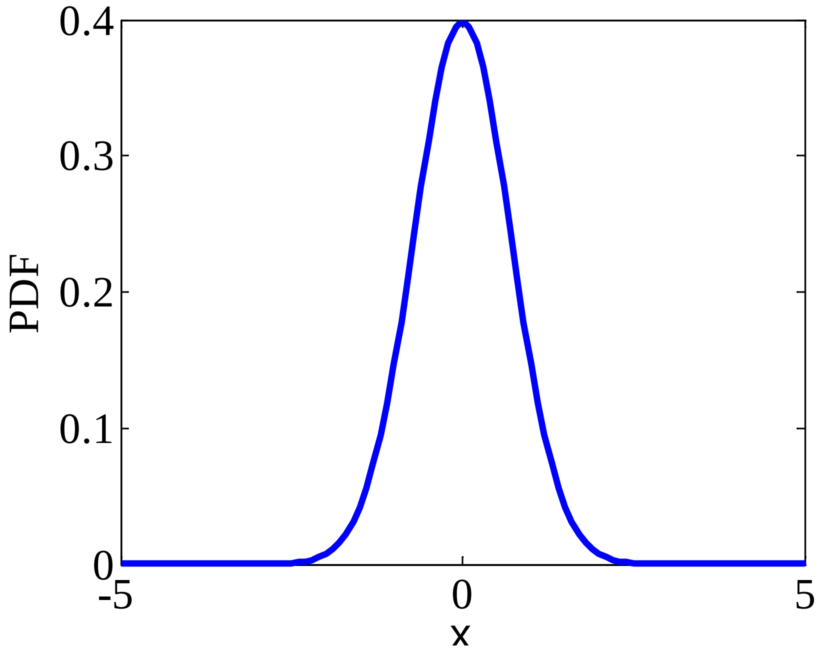
  - Variance : 2-order moment ($c = \mu$)

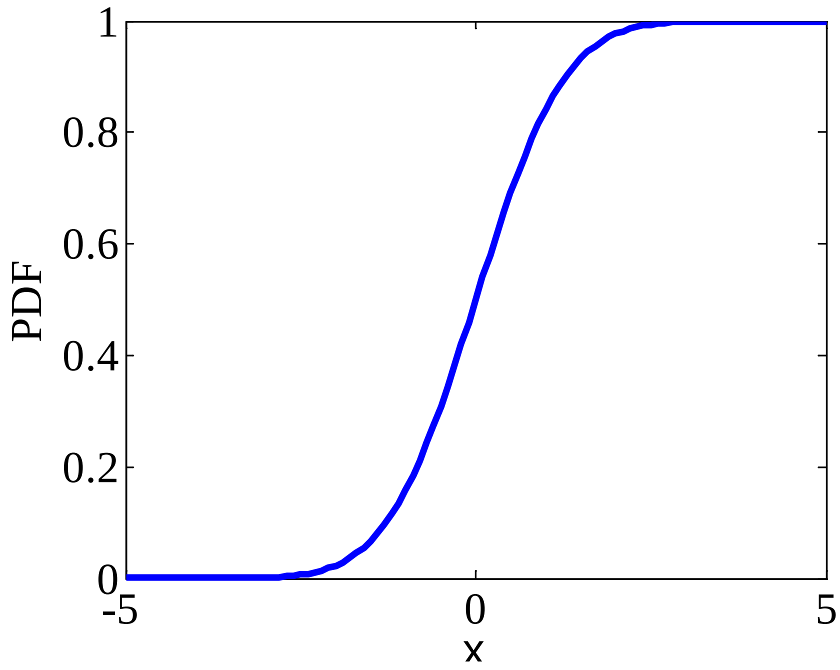
$$E[(X-\mu)^2] = \sigma^2 = m_2 = \int_{-\infty}^{\infty} (x-\mu)^2 f_X(x)dx$$
$$= E[X^2] - \mu^2$$

Normal (Gaussian) Distribution

- Standard normal distribution

$$g(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \qquad G(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{t^2}{2}\right) dt$$



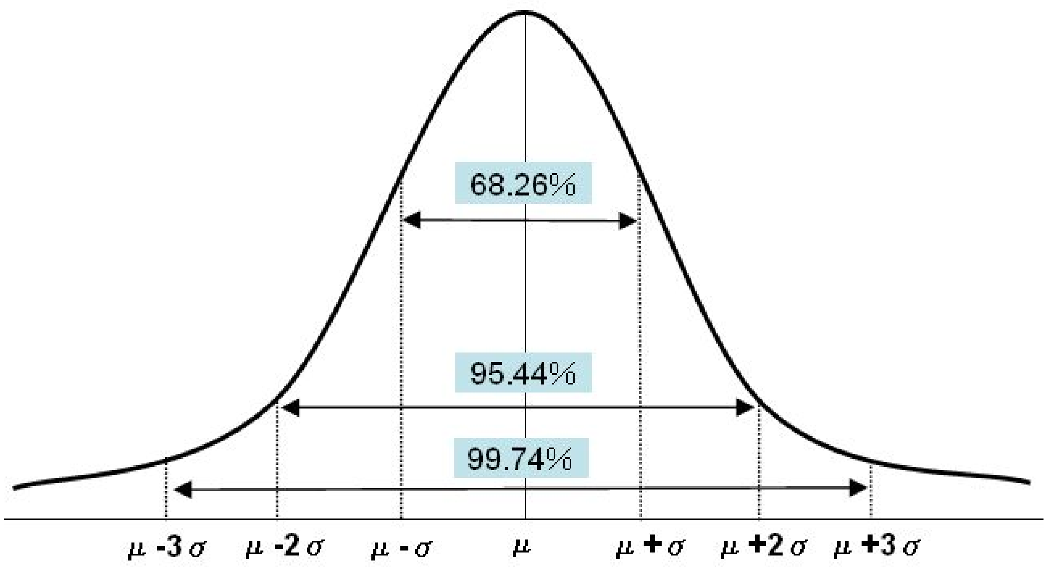


Normal (Gaussian) Distribution

- Normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) = \frac{1}{\sigma} g\left(\frac{x-\mu}{\sigma}\right)$$

$$F(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{x} \exp\left(-\frac{(t-\mu)^2}{2\sigma^2}\right) dt = G\left(\frac{x-\mu}{\sigma}\right)$$



Q Function and Error Function

- Q function: the *upper tail probability* of a standard normal distribution

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{t^2}{2}\right) dt = 1 - G(x)$$

- Error function:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^{x} \exp(-t^2)\, dt = 1 - 2Q(\sqrt{2}x)$$

$$Q(x) = \frac{1}{2} - \frac{1}{2}\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

Radio Propagation

- The variation can be categorized into two kinds:
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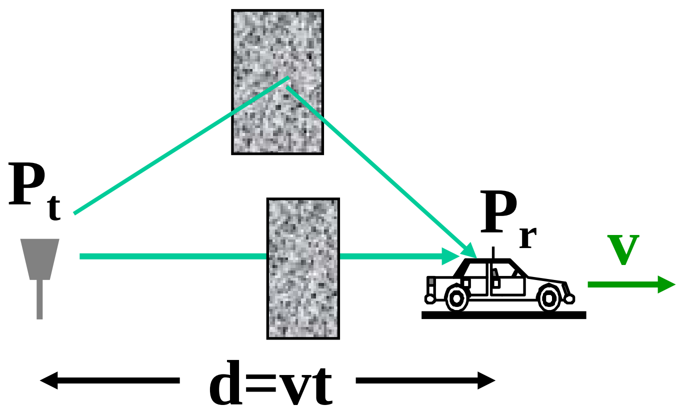
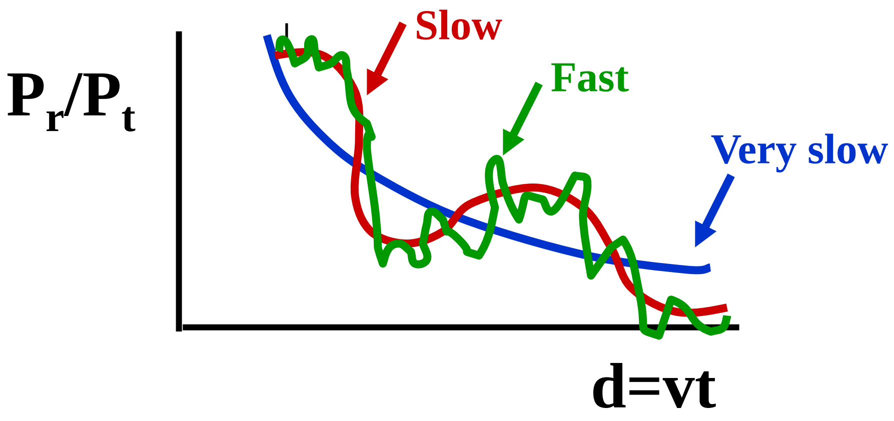
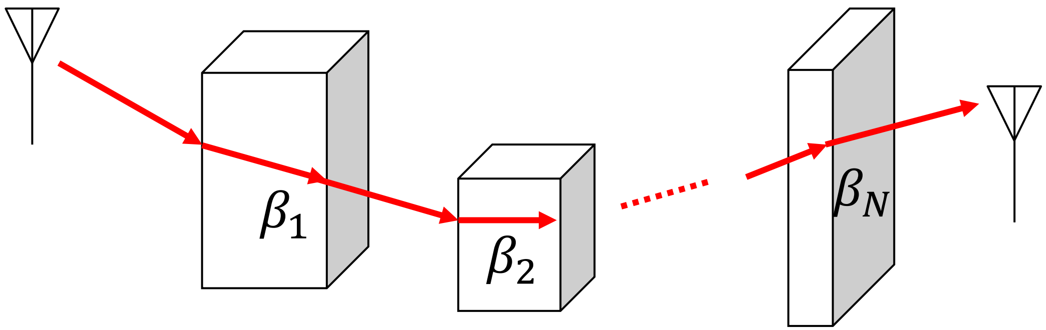
    - Shadowing (due to obstructions)
  - Small scale propagation effects: constructive or destructive addition of multipath signal components → multipath fading





Shadowing

- Signal path in terrain with several diffraction points adding extra attenuation to the path loss



- Total path loss

$$L_{\text{tot}} = L(d)\beta_1\beta_2\cdots\beta_N$$

$$L_{\text{tot}}[\text{dB}] = L(d)[\text{dB}] + \beta_1[\text{dB}] + \beta_2[\text{dB}] + \cdots + \beta_N\,[\text{dB}]$$

where $L(d)$ denotes the path loss in terms of d

- If $\beta_1, \beta_2, \cdots, \beta_N$ are considered random and independent, we should get a *normal distribution in the dB domain*

中心極限定理
(Central Limit Theorem)

Log-normal Distribution

- Let us consider $\psi = P_t/P_r$ where the received power is randomly varied due to shadowing
- $\psi_{\mathrm{dB}} = 10\log_{10}\psi$
- PDF of ψ_{dB}

$$f_{\Psi_{\mathrm{dB}}}(\psi_{\mathrm{dB}}) = \frac{1}{\sqrt{2\pi}\sigma_{\psi_{\mathrm{dB}}}}\exp\left[-\frac{\left(\psi_{\mathrm{dB}} - \mu_{\psi_{\mathrm{dB}}}\right)^2}{2\sigma_{\psi_{\mathrm{dB}}}^2}\right]$$

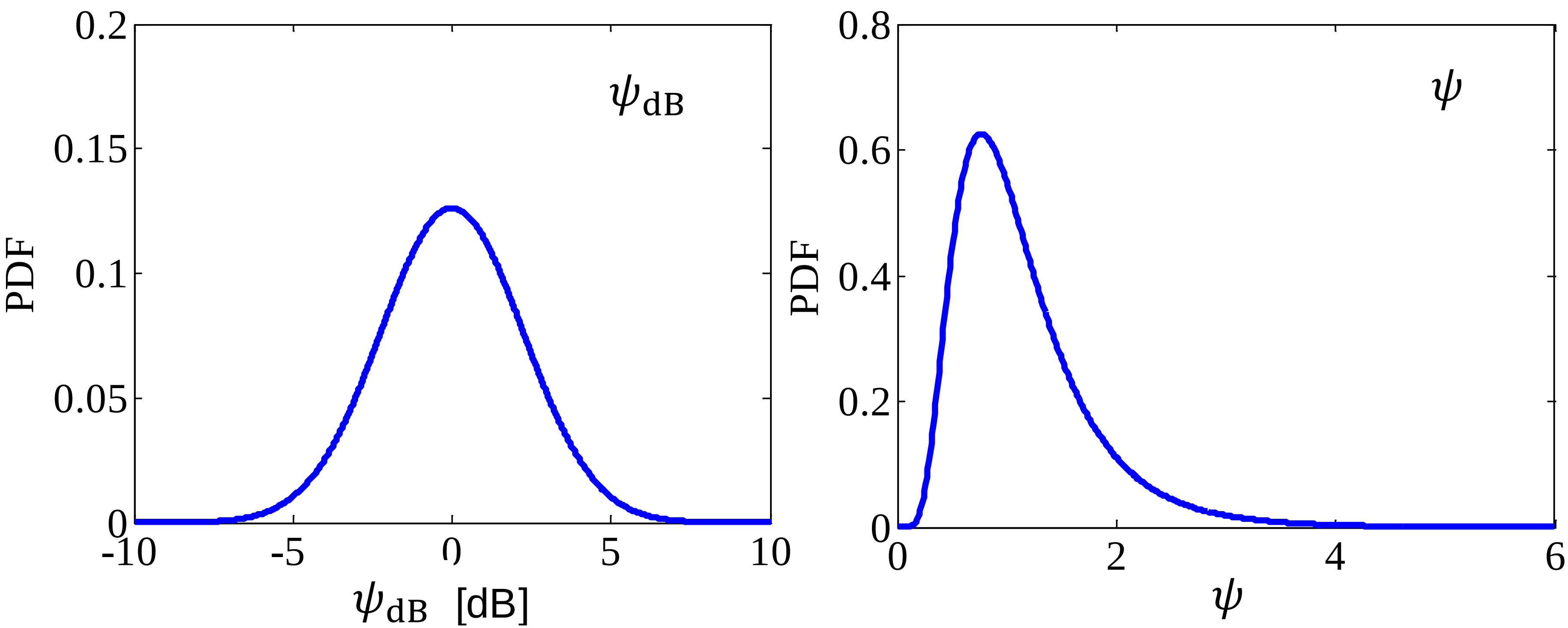
- PDF of ψ

$$f_{\Psi}(\psi) = \frac{\xi}{\sqrt{2\pi}\sigma_{\psi_{\mathrm{dB}}}\psi}\exp\left[-\frac{\left(10\log_{10}\psi - \mu_{\psi_{\mathrm{dB}}}\right)^2}{2\sigma_{\psi_{\mathrm{dB}}}^2}\right]$$

where $\xi = 10/\ln 10$

Log-normal Distribution

- For example, $\mu_{\psi_{\mathrm{dB}}} = 0$ [dB], $\sigma_{\psi_{\mathrm{dB}}} = 3$ [dB]



Analytical Model for Shadowing

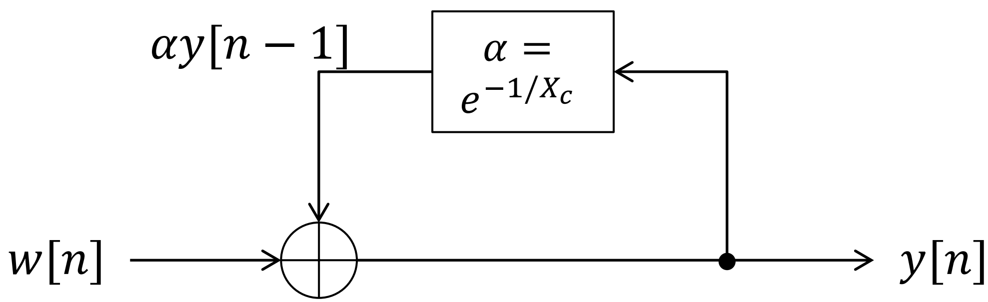
- Method to generate correlated shadowing process, from white Gaussian process

  Autocovariance:

$$A(\delta) = E\left[\left(\psi_{\mathrm{dB}}(d) - \mu_{\psi_{\mathrm{dB}}}\right)\left(\psi_{\mathrm{dB}}(d+\delta) - \mu_{\psi_{\mathrm{dB}}}\right)\right]$$
$$\sim \sigma^2_{\psi_{\mathrm{dB}}} e^{-\delta/X_c}$$

  where X_c denotes decorrelation distance
  (50m ~ 100m for outdoor system)

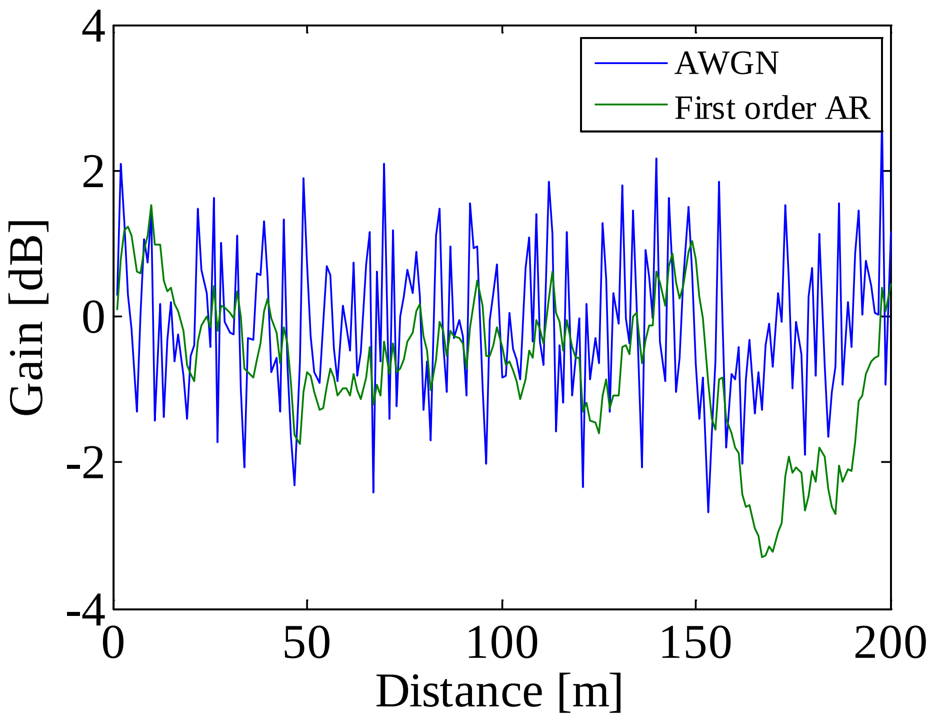
- First-order autoregressive correlation model

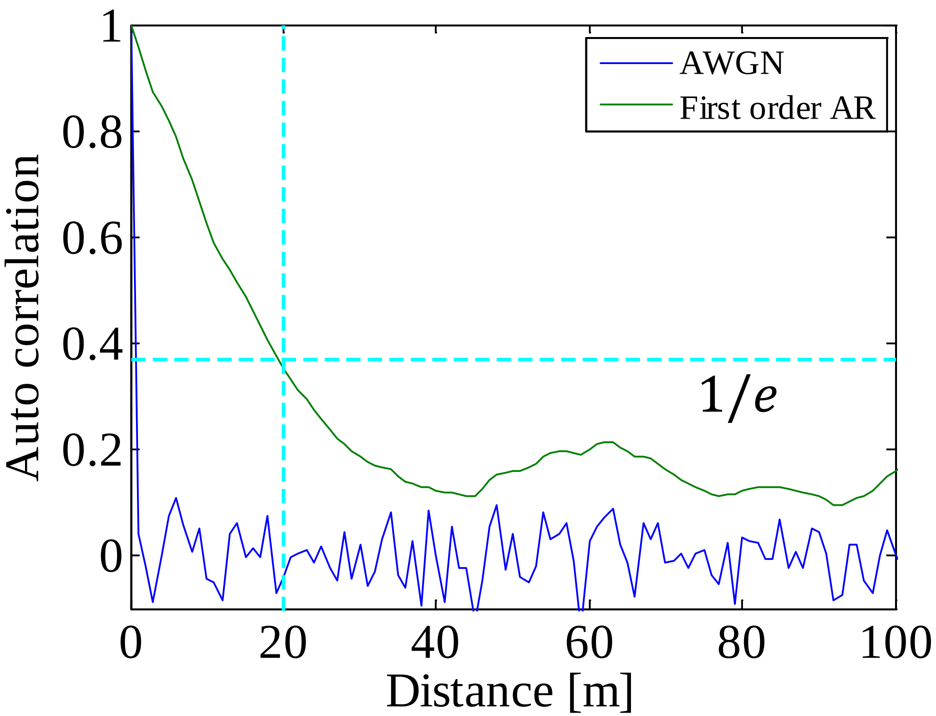


$$y[n] = \alpha y[n-1] + \underbrace{w[n]}_{\text{White Gaussian Noise}}$$

Analytical Model for Shadowing

- Example
  Plot the level for 0~200 m following the first-order autoregressive model when $X_c = 20$ m





Exercise

- Exercise 2-20

We will simulate the log-normal fading process over distance based on the autocorrelation model (pp. 34). The simulation first generates a white noise process and then passes it through a first order filter with a pole at $e^{-\delta/X_c}$. Assume $X_c = 20$m.

  - Plot the resulting log-normal fading process over a distance d ranging from 0 to 200m, sampling the process every meter. (It should be needed to normalize your plot about 0 dB) → See pp. 35
  - Plot the power spectrum
  - Plot the autocorrelation → See pp. 35
  - Compare above results with those of AWGN

- Simulation tool (Matlab or Scilab) should be used
- Report due: May 24

Combined Path Loss and Shadowing

- Combined expression of path loss

Pathloss + average shadowing

$$\frac{P_t}{P_r}\,[\mathrm{dB}] = L(d) + \psi_{\mathrm{dB}}$$

$$= 10\log_{10} K + 10\gamma \log_{10}\frac{d}{d_0} + \psi_{\mathrm{dB}}$$

Constant

Path loss exponent

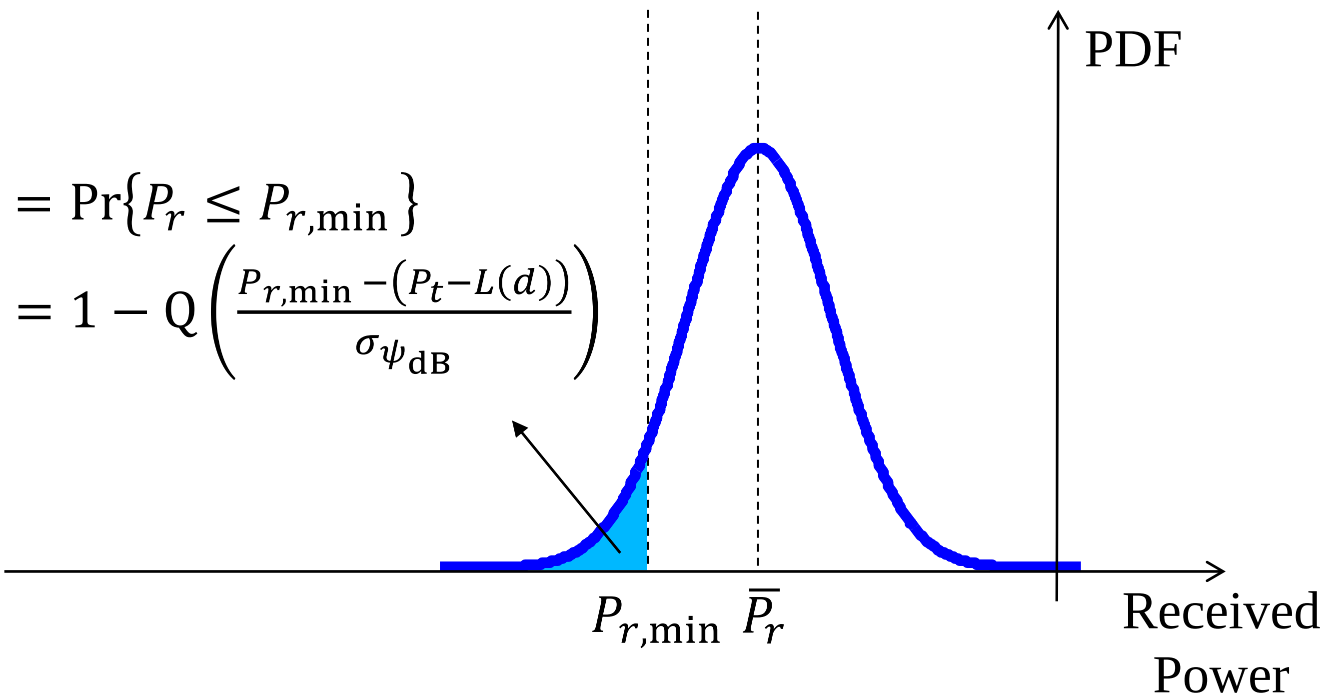
Gaussian random variable with zero mean and variance $\sigma^2_{\psi_{\mathrm{dB}}}$

Outage Probability

- The probability that the received power at a given distance d, $P_r(d)$ falls below the target minimum level

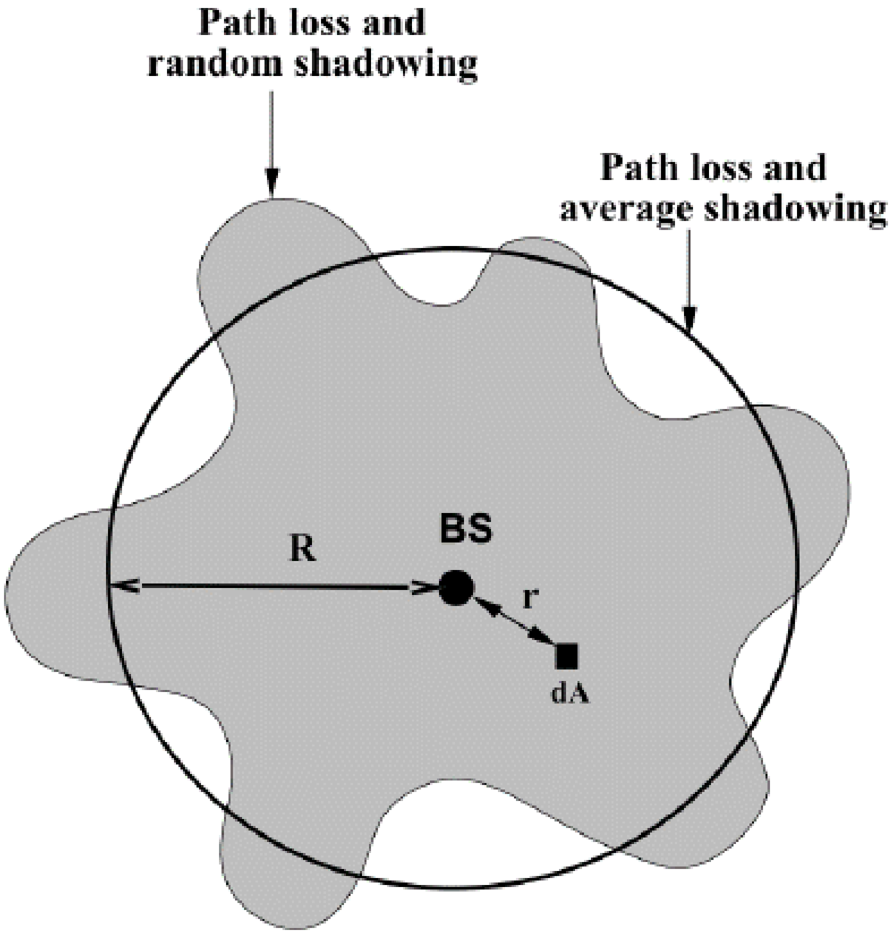
$$P_r(d)[\mathrm{dB}] = \underbrace{P_t[\mathrm{dB}] - L(d)}_{\text{mean received power at } d} + \psi_{\mathrm{dB}}$$

$$P_{\mathrm{out}} = \Pr\{P_r \le P_{r,\min}\} = 1 - \mathrm{Q}\left(\frac{P_{r,\min} - \left(P_t - L(d)\right)}{\sigma_{\psi_{\mathrm{dB}}}}\right)$$



Cell Coverage Area

- Random shadowing fluctuations result in “ameba-like shape”



Cell Coverage Area

- Expected percentage of locations within cell where the received power at these locations is above a given minimum
- Integration over all incremental areas satisfy the min. power requirement

$$C = \frac{1}{\pi R^2} \int_{\text{cell area}} P_A dA = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R P_A r dr d\theta$$

- Coverage probability is simply obtained from outage probability as

$$P_A = \Pr(P_r(r) > P_{\min})$$

$$= Q\left(\frac{P_{\min} - (P_t - L(r))}{\sigma_{\psi_{\text{dB}}}}\right) = 1 - P_{\text{out}}(P_{\min}, r)$$

Cell Coverage Area

- Received power due to path loss and average shadowing at the cell boundary equals to the minimum received power

$$\rightarrow P_{\min} = \bar{P}_r(R) = P_t - \left(10\log_{10} K - 10\gamma \log_{10}\frac{R}{d_0}\right)$$

remind $\boxed{P_r(r) = P_t - \left(10\log_{10} K - 10\gamma\log_{10}\frac{r}{d_0}\right) + \psi_{\mathrm{dB}}}$

- Find the coverage area for $\gamma = 2,4,6$ and $\sigma_{\psi_{\mathrm{dB}}} = 4,8,12$ dB

$$C = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R Q\left(\frac{P_{\min} - P_r(r)}{\sigma_{\psi_{\mathrm{dB}}}}\right) r\,dr\,d\theta = \frac{2}{R^2}\int_0^R Q\left(b\ln\frac{r}{R}\right) r\,dr$$

$$C = \frac{1}{2} + \exp\left[\frac{2}{b^2}\right] Q\left(\frac{2}{b}\right)$$

where $b = 10\gamma\log_{10} e / \sigma_{\psi_{\mathrm{dB}}}$

$\gamma \setminus \sigma_{\psi_{dB}}$	4	8	12
2	.77	.67	.63
4	.85	.77	.71
6	.90	.83	.77