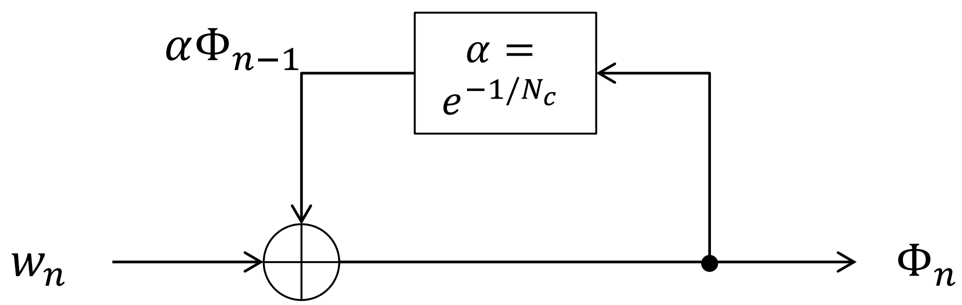
1-order Auto Regressive Process

- Method to generate correlated random process, from white Gaussian process
- First-order autoregressive correlation model



$$\Phi_n = \alpha\Phi_{n-1} + w_n$$

White Gaussian Noise
mean μ_w and variance σ_w^2

Coherence time $\boxed{T_c = \frac{N_c}{f_s}}$

1-order Auto Regressive Process

- Mean

$$\mu_\Phi = \frac{\mu_w}{1-\alpha} \qquad \because E[\Phi_n] = \alpha E[\Phi_{n-1}] + E[w_n]$$

- Variance

$$\sigma_\Phi^2 = \frac{\sigma_w^2}{1-\alpha^2} \qquad \because \Phi_n^2 = \alpha^2\Phi_{n-1}^2 + 2\Phi_{n-1}w_n + w_n^2$$

- Autocorrelation for $\mu_\Phi = 0$

$$A_{n_\tau} = E\left[\Phi_n\Phi_{n+n_\tau}\right] = \sigma_\Phi^2\alpha^{-|n_\tau|} = \sigma_\Phi^2 e^{-|n_\tau|/N_c}$$

$$\because \; \Phi_n\Phi_{n+n_\tau} = \alpha\Phi_{n-1}\Phi_{n+n_\tau} + w_n\Phi_{n+n_\tau}$$

1-order Auto Regressive Process

- Power Spectrum

$$\hat{P}(\omega) = \mathrm{DTFT}\left\{e^{-|n_\tau|/N_c}\right\} = \sum_{n_\tau=-\infty}^{\infty} \sigma_\Phi^2 e^{-|n_\tau|/N_c} e^{-\mathrm{j}\omega n_\tau} \quad \because \alpha = e^{-1/N_c}$$

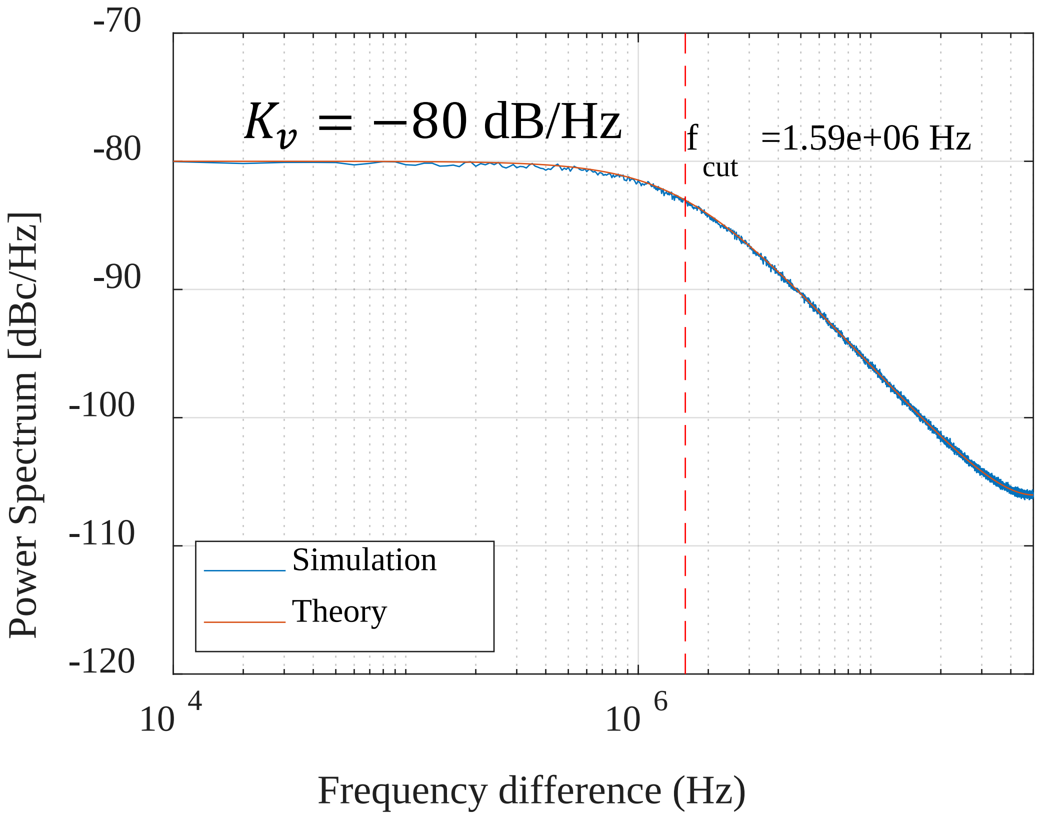
$$= \sigma_\Phi^2 \frac{1-\alpha^2}{1+\alpha^2-2\alpha\cos\omega}$$

$$P(f) = \sigma_\Phi^2 \frac{1-\alpha^2}{1+\alpha^2-2\alpha\cos 2\pi f} \cdot \frac{1}{f_s} \quad \text{[power/Hz]}$$

$$f_{\mathrm{cut}} = \cos^{-1}\left(\frac{4\alpha-1-\alpha^2}{2\alpha}\right)\frac{f_s}{2\pi}$$

Example

- Power Spectrum



$$\sigma_w^2 = 10^{\frac{K_v}{10}} f_s (1-\alpha)^2$$

$$f_{\text{cut}} = \cos^{-1}\left(\frac{4\alpha - 1 - \alpha^2}{2\alpha}\right)\frac{f_s}{2\pi}$$